A Closed-Form Solution for the Uniform Sampling of the Epipolar Line via Non-Uniform Depth Sampling

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Given two camera calibrations, this report presents a closed form algorithm that computes a sequence of 3D points such that they all project to a single location on one camera and that their projection forms a uniformly sampled line on the other camera.

Let a camera be parametrized by its intrinsic parameters $\mathbf{K}$, and extrinsic parameters of rotation matrix $\mathbf{R}$ and camera center $\mathbf{C}$. The projection of a 3D point $\mathbf{X}$ is defined as

$$\lambda \mathbf{x} = \mathbf{K}\mathbf{R}(\mathbf{X} - \mathbf{C}) \tag{1}$$

with $\mathbf{x} = [x\ y\ 1]^T$ its image coordinates and λ its depth. Then, the back-projected ray emanating from point $\mathbf{x} = [x\ y\ 1]^T$ is parametrized in terms of the depth variable λ as:

$$\mathbf{X}(\lambda) = \lambda \mathbf{R}^T \mathbf{K}^{-1} \mathbf{x} + \mathbf{C} \quad . \tag{2}$$

Given two camera calibration matrices $P_0 = (\mathbf{K}_0, \mathbf{R}_0, \mathbf{C}_0)$ and $P_1 = (\mathbf{K}_1, \mathbf{R}_1, \mathbf{C}_1)$, we would like to sample the back-projected line $\mathbf{X}(\lambda)$ so that the projected samples on camera P_1 are uniformly separated (see Figure 1).

The two points separated by depth $d\lambda$ on $\mathbf{X}(\lambda)$ is equal to:

$$\mathbf{X}(\lambda) = \lambda \mathbf{R}_0^T \mathbf{K}_0^{-1} \mathbf{x} + \mathbf{C}_0 \tag{3}$$

$$\mathbf{X}(\lambda + d\lambda) = (\lambda + d\lambda) \mathbf{R}_0^T \mathbf{K}_0^{-1} \mathbf{x} + \mathbf{C}_0 \quad . \tag{4}$$

Then, the projection of these points on image P_1 are

$$\omega \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \lambda \mathbf{K}_1 \mathbf{R}_1 \mathbf{R}_0^T \mathbf{K}_0^{-1} \mathbf{x} + \mathbf{K}_1 \mathbf{R}_1 (\mathbf{C}_0 - \mathbf{C}_1) \tag{5}$$

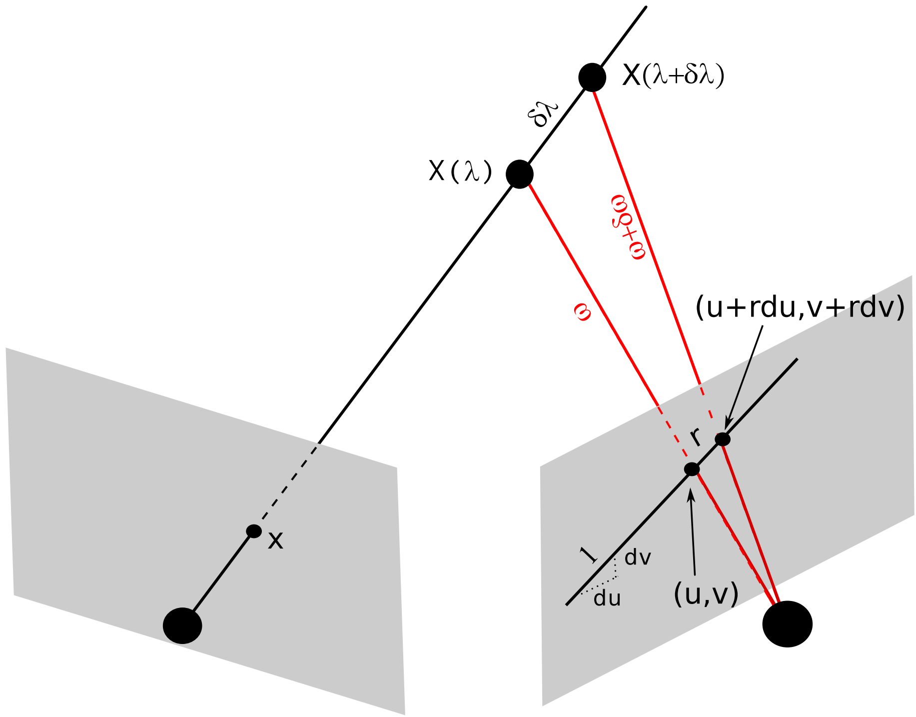

Figure 1: **Uniform Step Sampling Framework**

and

$$(\omega + d\omega)\begin{bmatrix} u + rdu \\ v + rdv \\ 1 \end{bmatrix} = (\lambda + d\lambda)\mathbf{K}_1\mathbf{R}_1\mathbf{R}_0^T\mathbf{K}_0^{-1}\mathbf{x} + \mathbf{K}_1\mathbf{R}_1(\mathbf{C}_0 - \mathbf{C}_1) \tag{6}$$

with (u, v) and ω the coordinates and depth of $\mathbf{X}(\lambda)$ on image P_1 respectively, $w + d\omega$ the depth of the point $\mathbf{X}(\lambda + d\lambda)$, (du, dv) the slope of the epipolar line and r is the sampling resolution.

The slope of the epipolar line can be found by projecting any two points $\mathbf{X}(\lambda_{min})$ and $\mathbf{X}(\lambda_{max})$ onto P_1. Let the coordinates of the projections are (u_{min}, v_{min}) and (u_{max}, v_{max}). The slope is then computed as:

$$dl = \sqrt{(u_{max} - u_{min})^2 + (v_{max} - v_{min})^2}$$
$$du = (u_{max} - u_{min})/dl \tag{7}$$
$$dv = (v_{max} - v_{min})/dl \quad .$$

Let

$$\begin{aligned} \mathbf{a} &= \mathbf{K}_1\mathbf{R}_1\mathbf{R}_0^T\mathbf{K}_0^{-1}\mathbf{x} \\ \mathbf{b} &= \mathbf{K}_1\mathbf{R}_1(\mathbf{C}_0 - \mathbf{C}_1) \end{aligned} \tag{8}$$

and rewrite Equations 5 and 6 as

$$\omega \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \lambda \mathbf{a} + \mathbf{b} \tag{9}$$

and

$$(\omega + d\omega) \begin{bmatrix} u + rdu \\ v + rdv \\ 1 \end{bmatrix} = (\lambda + d\lambda)\mathbf{a} + \mathbf{b} \ . \tag{10}$$

Rearranging Equations 9 and 10, one can see the relationship between the changes in the depths for two views as:

$$\mathbf{a} d\lambda = \omega \begin{bmatrix} rdu \\ rdv \\ 0 \end{bmatrix} + d\omega \begin{bmatrix} u + rdu \\ v + rdv \\ 1 \end{bmatrix} \tag{11}$$

If we denote the i^{th} element of vector $\mathbf{a}$ with a_i and expand the third row of the Equation 11,

$$a_2 d\lambda = d\omega \ , \tag{12}$$

the first two rows of Equation 11, after some rearranging, becomes

$$\begin{aligned} d\lambda &= \frac{\omega r du}{a_0 - a_2(u + rdu)} \\ d\lambda &= \frac{\omega r dv}{a_1 - a_2(v + rdv)} \ . \end{aligned} \tag{13}$$

Equation 13 is the update in the depth with respect to camera P_0 that one must make in order to move r pixels in the direction of (du, dv) on camera P_1 where (u, v) and ω is the current projection and current depth with respect to camera P_1. (see Figure 1). Thus, the sequence of 3D points that will have a uniform sampling projection can be computed by iterating this process.

The pseudo-code that computes the sampling locations and the depths associated with these points are given in Algorithm 1. A Matlab implementation for the algorithm is given in [Tola, 2010].

References

[Tola, 2010] Tola, E. (2010). Uniform sampling of epipolar line via non-uniform depth sampling - http://cvlab.epfl.ch/˜tola/src/episample.

Algorithm 1: Non-uniform Space Sampling

Require: Camera Parameters: $(\mathbf{K}_0, \mathbf{R}_0, \mathbf{C}_0)$ and $(\mathbf{K}_1, \mathbf{R}_1, \mathbf{C}_1)$
Require: Point location $\mathbf{x}$
Require: Depth range $(\lambda_{min}, \lambda_{dmax})$
Require: Sampling resolution r

$\mathbf{a} \Leftarrow \mathbf{K}_1\mathbf{R}_1\mathbf{R}_0^T\mathbf{K}_0^{-1}\mathbf{x}$
$\mathbf{b} \Leftarrow \mathbf{K}_1\mathbf{R}_1(\mathbf{C}_0 - \mathbf{C}_1)$
$u_{min} \Leftarrow \frac{\lambda_{min}a_0+b_0}{\lambda_{min}a_2+b_2}, \quad v_{min} \Leftarrow \frac{\lambda_{min}a_1+b_1}{\lambda_{min}a_2+b_2}$
$u_{max} \Leftarrow \frac{\lambda_{max}a_0+b_0}{\lambda_{max}a_2+b_2}, \quad v_{max} \Leftarrow \frac{\lambda_{max}a_1+b_1}{\lambda_{max}a_2+b_2}$
$dl \Leftarrow \sqrt{(u_{max} - u_{min})^2 + (v_{max} - v_{min})^2}$
$du \Leftarrow (u_{max} - u_{min})/dl, \; dv \Leftarrow (v_{max} - v_{min})/dl$
$\omega_{min} \Leftarrow \lambda_{min}a_2 + b_2$
$\omega \Leftarrow \omega_{min}$
$\lambda \Leftarrow \lambda_{min}$
$(u, v) =\Leftarrow (u_{min}, v_{min})$
while $\lambda < \lambda_{max}$ **do**
    if $|du|$ not equal to 0 **then**
        $d\lambda \leftarrow \frac{\omega r du}{a_0 - a_2(u + rdu)}$
    else
        $d\lambda \leftarrow \frac{\omega r dv}{a_1 - a_2(v + rdv)}$
    end if
    $\lambda \leftarrow \lambda + d\lambda$
    $(u, v) \leftarrow (u + rdu, v + rdv)$
    $X \leftarrow \lambda\mathbf{R}^T\mathbf{K}^{-1}\mathbf{x} + \mathbf{C}$
    $\omega \leftarrow \lambda a_2 + b_2$
end while