

IMAGE FORMATION

From 3-D physical world to 2-D image:

- Analog vs digital images
- Camera geometry
- Radiometry and sensing

ANALOG IMAGES

An image on film can be understood a two-dimensional light intensity function $f(x,y)$ where:

- x and y are spatial coordinates
 - The value of f at any point (x, y) is proportional to the brightness or gray value of the image at that point.
- Cannot be stored as such on a digital computer.

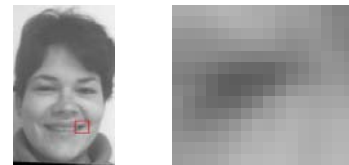
DIGITAL IMAGES

A digitized image is one in which:

- Spatial and grayscale values have been made discrete.
- Intensities measured across a regularly spaced grid in x and y directions are sampled to
 - 8 bits (256 values) per point for black and white,
 - 3x8 bits per point for color images.

They are stored as a two dimensional arrays of gray-level values. The array elements are called pixels and identified by their x, y coordinates.

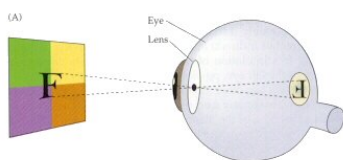
PIXELS



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236 124 361 120 360 360 371 373 371 360 359 337 335
332 345 336 330 351 349 353 354 350 341 363 363 339 351
345 349 349 349 349 333 335 345 345 345 345 346 346 346
346 345 341 345 345 345 341 336 336 336 336 336 336 333
333 333 329 329 329 326 340 340 340 336 336 336 332 332
335 333 336 336 336 336 333 333 333 337 347 346 341 329 323
337 339 339 339 339 339 339 339 339 339 339 339 341 347 347 345
336 334 335 336 336 336 336 336 337 337 337 341 341 347 339 332
352 352 337 334 333 338 335 338 337 337 339 339 350 357 359
359 357 357 359 359 351 351 351 351 352 356 347 356 363
365 365 363 363 363 366 336 333 333 336 365 365 354 363
366 366 370 368 368 368 370 370 346 346 347 348 352 359
368 370 370 370 370 370 370 370 370 370 370 370 370 370 370
363 370 370 370 370 370 370 370 370 370 370 370 370 370 370
361 362 360 370 360 360 360 360 360 370 370 370 360 360
    
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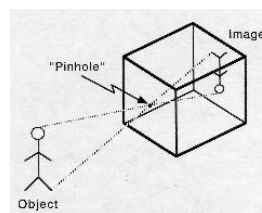
IMAGE FORMATION



Projection from surfaces to 2-D sensor.

- Where: Geometry
- How bright: Radiometry
- Stored how: Sensing

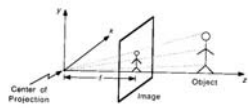
PINHOLE CAMERA MODEL



Idealized model that defines perspective projection:

- All rays go through a hole and form a star of lines
- The hole acts as a selector of rays that allows the formation of an inverted image.

CAMERA GEOMETRY



$$u = f \frac{x}{z}$$

$$v = f \frac{y}{z}$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} \text{ with } u = \frac{U}{W}, y = \frac{V}{W}$$

$$\begin{bmatrix} \lambda X_c \\ \lambda Y_c \\ \lambda Z_c \\ \lambda \end{bmatrix} = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} U \\ V \\ W \end{bmatrix} = P \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \text{ with } P = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$$

CAMERA CALIBRATION

Internal Parameters:

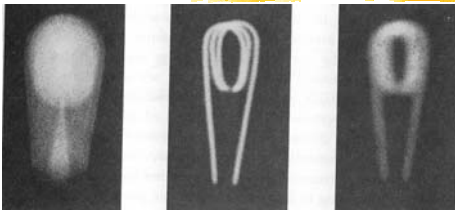
- Horizontal and vertical scaling (2)
- Principal points (2)
- Skew of the axis (1)

External Parameters:

- Rotations (3)
- Translations (3)

→ **11 free parameters.**

LIMITATIONS

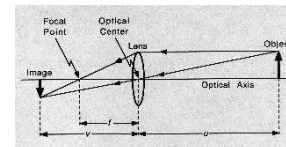


Idealization because the hole cannot be infinitely small

- Image would be infinitely dim
- Diffraction effects

→ Use of Lenses.

IMAGING WITH A LENS



Thin lens Equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

→ Lens with focal distance f equivalent to pinhole camera with similar focal distance but larger aperture.

DISTORTIONS



The lens is not exactly a "thin lens:"

- Different wave lengths refracted differently
- Barrel Distortion

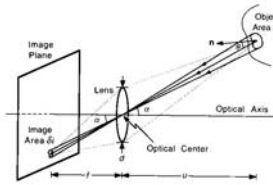
UNDISTORTING



Once the image is undistorted, the camera projection can be formulated as a projective transform.

→ The pinhole camera model applies.

RADIOMETRY



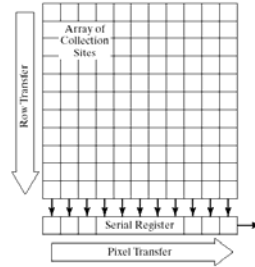
Scene Radiance: Amount of light radiation from a surface point (Watt / m² / Steradian)

Image Irradiance: Amount of light incident at the image of the surface point. (Watt / m²)

Fundamental Radiometric Equation:

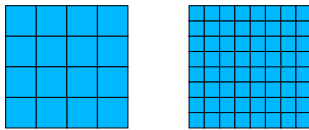
$$Irr = \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4(\alpha) Rad$$

CHARGE COUPLED DEVICE



- Photons free up electrons that are then captured by a potential well.
- Charges are transferred row by row wise to a register.
- Pixel values are read from the register.

SENSING



Conversion of the "optical image" into an "electrical image":

$$E(x, y) = \iint Irrad(x, y, t, \lambda) s(\lambda) \tau(t - t_0) d\lambda dt$$

$$I(m, n) = Quantize(\iint E(x, y) \omega(x - m, y - n) dx dy)$$

→ Quantization in

- Time
- Space

IN SHORT

- Camera geometry can be modeled in terms of the pinhole camera model, which is linear in projective space.
- Image radiance is roughly proportional to surface radiance and the two can be used interchangeably for our purposes.