SHAPE FROM X

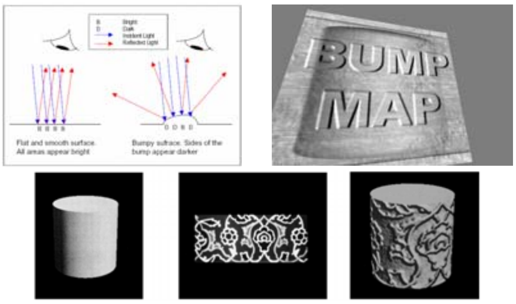
One image:
- Texture
- **Shading**

Two images or more:
- Stereo
- Contours
- Motion

SHADING

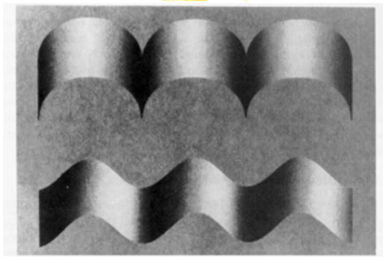
- Shading models
- Shape from shading
  - Quantitative
  - Qualitative

BUMP MAPPING



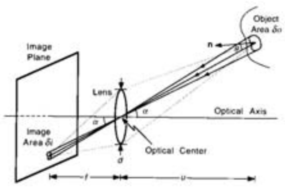
Simple mesh + 2D bump map = Complex looking object

BOUNDARY CONDITIONS



Shading gives information about surface normals
→ Boundary conditions required for a complete interpretation.

FUNDAMENTAL RADIOMETRIC EQUATION

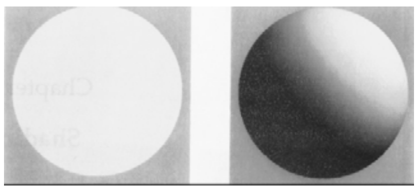


$$Irr = \frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha) Rad$$

→ When using either an orthographic or a photometrically calibrated camera:

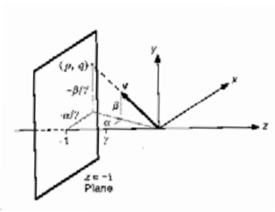
$$Irr \propto Rad$$

LAMBERTIAN HALF-SPHERE



Gray level changes are interpreted as changes in the direction of the surface normals

GRADIENT SPACE



Unit vector: $[\alpha,\beta,\gamma]$

Can be represented by (p,q,-1) where:

$$p = -\frac{\alpha}{\gamma}$$

$$q = -\frac{\beta}{\gamma}$$

SURFACE NORMALS

Imaged surface:

$$z = F(x, y)$$

- Normal vector:

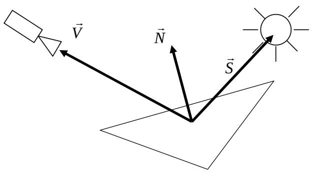
$$\left[\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y}, -1\right]$$

- Tangent vectors:

$$\left[1, 0, \frac{\delta F}{\delta x}\right]$$

$$\left[0, 1, \frac{\delta F}{\delta y}\right]$$

SIMPLIFYING ASSUMPTIONS



- The illumination sources are distant from the imaged surfaces
- Secondary illumination is not significant
- There are no cast shadows

REFLECTANCE MAP

Reflectance:

Proportion of the light reflected in a given direction

Albedo:

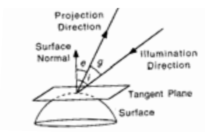
Fraction of the light incident on the surface that is reflected over all directions.

→**For constant albedo:**

$$Irr(u, v) = \text{Ref}(p(u, v), q(u, v))$$

Image Irradiance Equation

ANGLES

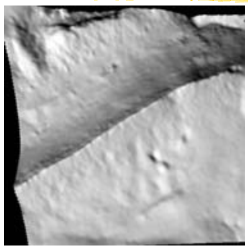


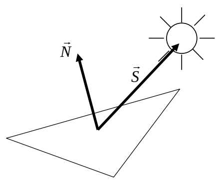
Angle of incidence: angle between the surface normal and the direction of the incident light ray.

Angle of emittance: angle between emitted light ray and surface normal.

Phase angle: angle between incident and emitted light ray.

LAMBERTIAN SURFACE

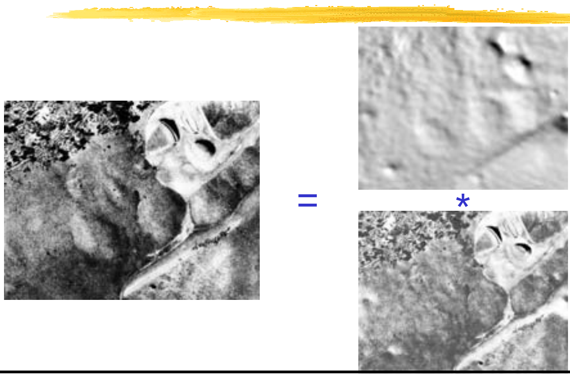




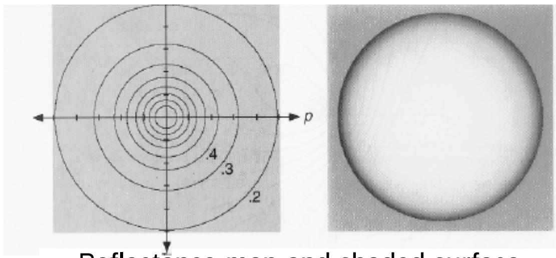
$$I = \alpha(\vec{N} \bullet \vec{S})$$

Perfectly matte surface: Radiance depends only on angle of incidence and not on viewing direction.

LAMBERTIAN SURFACE

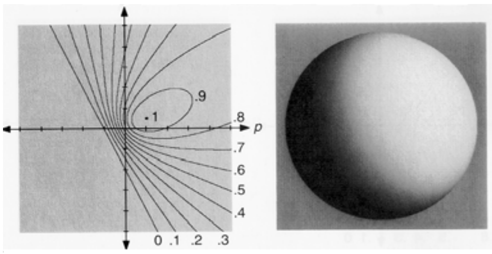


LAMBERTIAN REFLECTANCE MAP



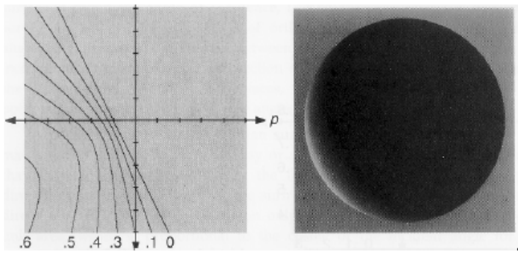
Reflectance map and shaded surface for lambertian surface illuminated in the direction [0 0 -1]

LAMBERTIAN REFLECTANCE MAP



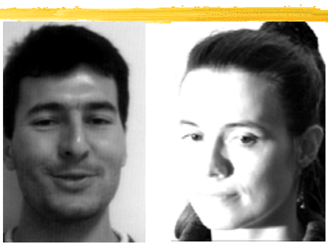
Reflectance map and shaded surface for lambertian surface illuminated in the direction [-1 -0.5 1]

LAMBERTIAN REFLECTANCE MAP



Reflectance map and shaded surface for lambertian surface illuminated in the direction [1 0.5 -1]

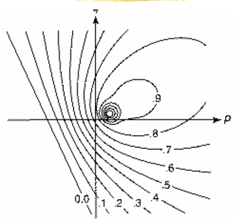
SPECULAR SURFACE



Perfect mirror: Reflects light only when i=e and i+e=g.

→ In practice mixture of specular and Lambertian.

LAMBERTIAN+SPECULAR REFLECTANCE MAP



Weighted average of the individual diffuse- and specular-component of glossy white paint.

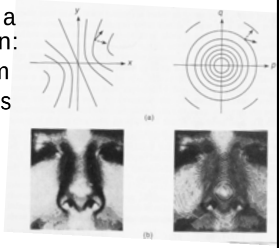
QUANTITATIVE SHAPE RECOVERY

- Characteristic strip method
- Variational methods
- Photometric stereo

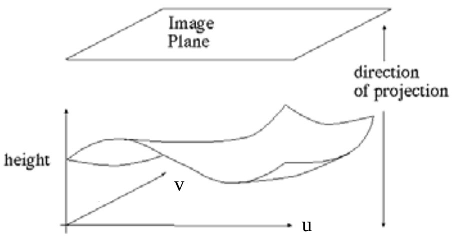
CHARACTERISTIC STRIP

Assuming constant albedo and a known light source direction:

1. Find a brightness extremum
2. Find iso-brightness contours
3. Infer surface normals
4. Iterate



DEPTH MAP MODEL



$z = f(u,v)$ Surface is set of points $\{u, v, f(u,v)\}$

Forsyth & Ponce

NORMALS

Surface normal can be written as

$$\frac{1}{\sqrt{1+p^2+q^2}}\begin{bmatrix} p \\ q \\ -1 \end{bmatrix}$$

where

$$p = \frac{\partial f}{\partial u} \text{ and } q = \frac{\partial f}{\partial v}.$$

$$\Rightarrow \frac{\partial p}{\partial v} = \frac{\partial q}{\partial u}$$

VARIATIONAL METHODS

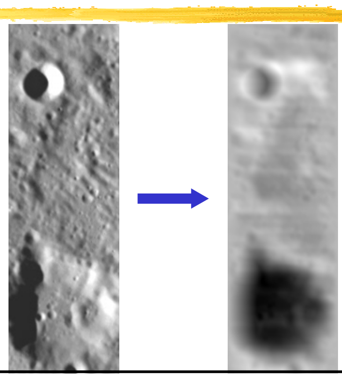
Minimize:

$$\iint \left\{ \begin{array}{l} [Irrad(u,v) - \mathrm{Ref}(\frac{\delta f}{\delta u}, \frac{\delta f}{\delta v})]^2 \\ + \lambda \left[\left[\frac{\delta^2 f}{\delta u^2} \right]^2 + 2\left[\frac{\delta^2 f}{\delta u \delta v} \right]^2 + \left[\frac{\delta^2 f}{\delta v^2} \right]^2 \right] \end{array} \right\} dudv$$

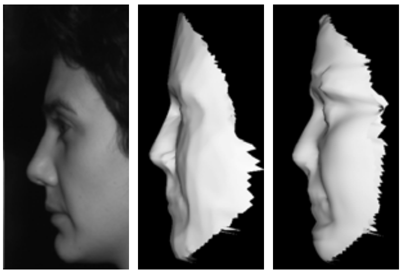
or:

$$\iint \left\{ \begin{array}{l} [Irrad(u,v) - \mathrm{Ref}(p,q)]^2 \\ + \lambda \left[\left[\frac{\delta p}{\delta u} \right]^2 + \left[\frac{\delta p}{\delta v} \right]^2 + \left[\frac{\delta q}{\delta u} \right]^2 + \left[\frac{\delta q}{\delta v} \right]^2 \right] \\ + \mu \left[\frac{\delta p}{\delta v} - \frac{\delta q}{\delta u} \right]^2 \end{array} \right\} dudv$$

MOONSCAPE

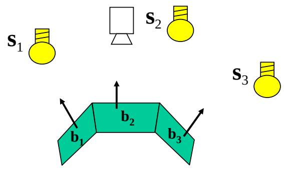


REFINING A STEREO RESULT



Stereo Shading

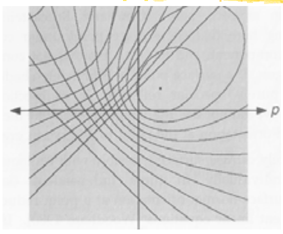
PHOTOMETRIC STEREO

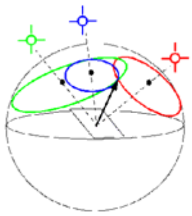


Given multiple images of the same surface under different known lighting conditions, can we recover the surface shape?

Yes! (Woodham, 1978)

PHOTOMETRIC STEREO





1. Take several images under different lighting conditions
2. Infer the orientation from the changes in illumination

ALGEBRAIC FORMULATION

Lambertian model: $I = \alpha(\vec{\mathbf{N}} \bullet \vec{\mathbf{L}}) = \vec{\mathbf{L}} \bullet \vec{\mathbf{M}}$

Three light sources:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \vec{\mathbf{L}}_1 \\ \vec{\mathbf{L}}_2 \\ \vec{\mathbf{L}}_3 \end{bmatrix} \vec{\mathbf{M}}$$

$$\vec{\mathbf{N}} = \frac{\vec{\mathbf{M}}}{\left\|\vec{\mathbf{M}}\right\|}$$

$$\alpha = \left\|\vec{\mathbf{M}}\right\|$$

ADDITIONAL LIGHTS

Over-constrained problem:

$$\mathbf{I} = \mathbf{L}\vec{\mathbf{M}}, \text{ with } \mathbf{I} = \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} \text{ and } \mathbf{L} = \begin{bmatrix} \vec{\mathbf{L}}_1 \\ \vdots \\ \vec{\mathbf{L}}_n \end{bmatrix}$$

$$\Rightarrow \mathbf{L}^t\mathbf{L}\vec{\mathbf{M}} = \mathbf{L}^t\mathbf{I} \text{ (Least-squares solution)}$$

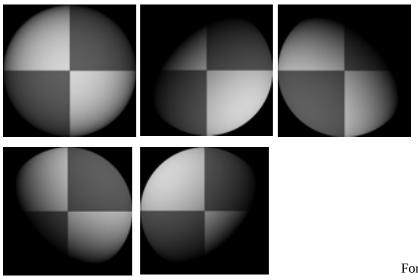
SHADOWS

- Shadowed pixels for a given light source position are outliers.
- Premultiplying by a thresholded weight matrix eliminates their contributions.

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \mathbf{I} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \mathbf{LM}$$

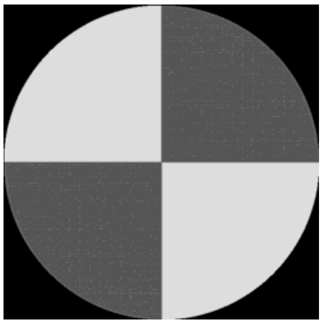
SYNTHETIC SPHERE IMAGES

Five different lighting conditions

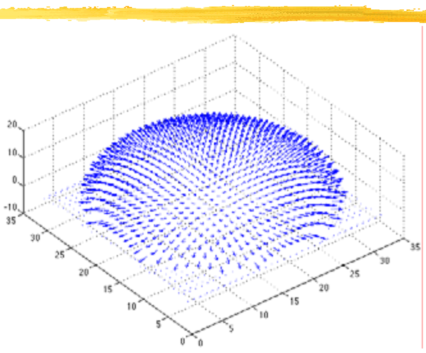


Forsyth & Ponce

RECOVERED ALBEDO



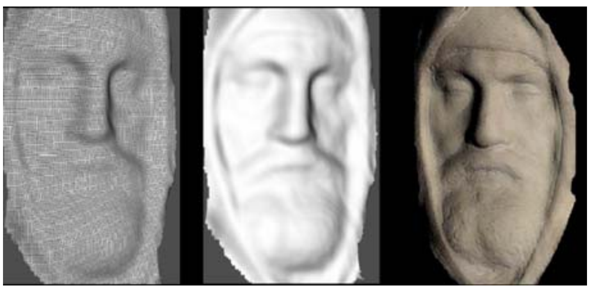
RECOVERED SURFACE NORMALS



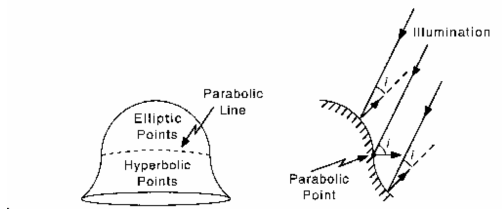
VIRTUOSO

One camera and five light sources

DELIGHTED TEXTURE MAPS

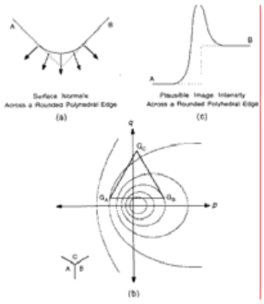


QUALITATIVE SHAPE RECOVERY



For smooth shapes, “highlight and shadows cling” to parabolic lines.

ARTIFACTS



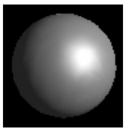
Rounding the edges of a polyhedral shape may lead to counterintuitive intensity profiles:

Instead of the expected step profile of (b) one finds the peaked profile of (c).

SECONDARY ILLUMINATION

Reflections produce indirect lighting.

Unique light source assumption doe not allow correct albedo recovery.



Recovered shading

Recovered albedoes

LIMITATIONS

- Constant albedo
- Inter-reflections
- Shadows

--> Shape from shading useful in conjunction with other sources of information but rarely by itself.