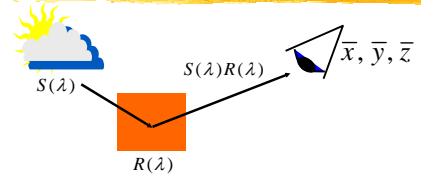


## COLOR



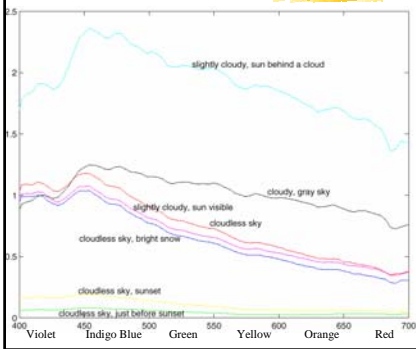
- Physics of color.
- Human perception.
- Digital representation.

## CAUSES OF COLOR



- Emission in different amounts at different wavelengths.
  - Absorption and reemission at different wavelengths.
  - Differential reflection
  - Differential refraction
- Light is rarely monochromatic.

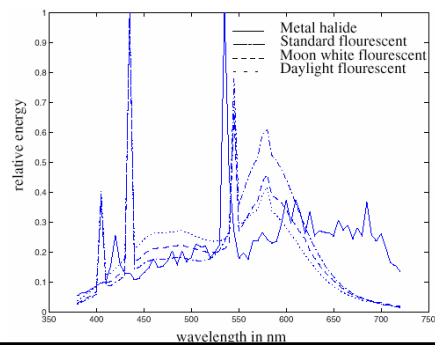
## RELATIVE SPECTRAL POWER OF SUNLIGHT



Color names on the horizontal axis are the names used for monochromatic light of the corresponding wavelength  
→ Colors of the rainbow.

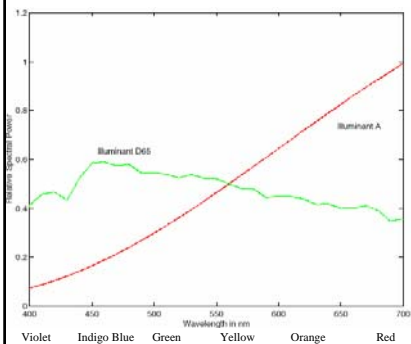
J. Parkkinen and P. Silfsten

## SPECTRAL POWER OF ARTIFICIAL ILLUMINANTS



H.Sugiura.

## SUNLIGHT VERSUS OF INCANDESCENT



- Green:
- Sunlight
- Red:
- Incandescent light

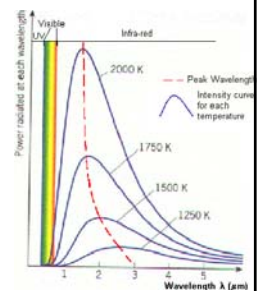
## BLACK BODY RADIATOR

The spectral power distribution of light leaving this object is a simple function of temperature

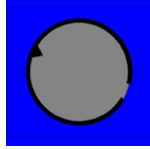
$$E(\lambda) \propto \left(\frac{1}{\lambda^5}\right) \left(\frac{1}{\exp(hc/k\lambda T) - 1}\right)$$

Color temperature: Temperature of a black body that would look the same

Example: The sun (5000 K) and the stars.



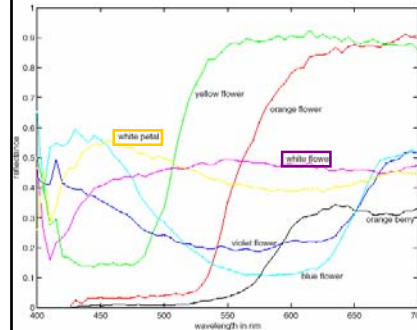
## BLACK BODY APPROXIMATION



Simplest way to construct a hot body with near-zero albedo:

- Build a hollow metal object with tiny hole in it.
  - Look at the hole.
- Very little of the light entering through the hole reaches the eye.

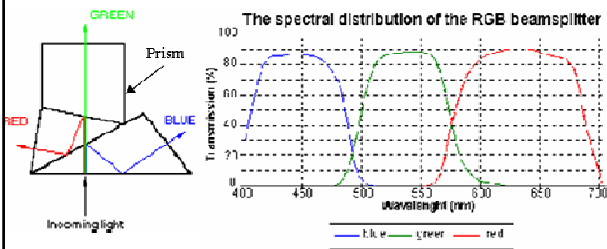
## SPECTRAL ALBEDOES FOR DIFFERENT FLOWERS



Different colors typically have different spectral albedoes, but different spectral albedoes may result in the same perceived color.

E. Koivisto.

## BEAM SPLITTER



## RADIOMETRY FOR COLOR

All definitions become "per unit wavelength"  
 All units become "per unit wavelength"  
 All terms become "spectral"

- Radiance becomes spectral radiance
- Radiosity becomes spectral radiosity

## HUMAN PERCEPTION

The sensation of color is caused by the brain in response to the presence/absence of light at various wavelengths.

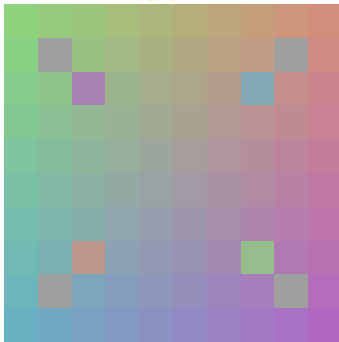
However, the perception of color can also result from:

- Pressure on the eyelids
- Dreaming, hallucinations
- Other less allowable means

## IMPORTANCE OF CONTEXT



## HOW COLORFUL IS THIS?



## STATE OF MIND



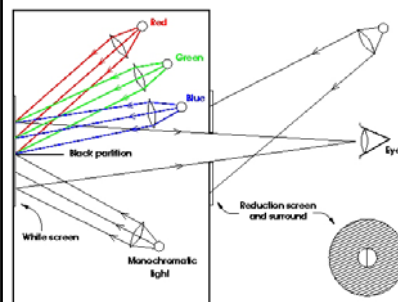
### Stroop Effect:

- Try quickly naming the colors of the letters in the middle column.
- Most people find this hard!

## SPECIFYING COLORS NUMERICALLY

- Few (about 10) color names are widely recognized by speakers of any language.  
→ Must provide a common vocabulary.
- Color reproduction problems are created by the prevalence of digital imaging  
→ Ensure that everyone sees the same color
- Accurate color reproduction is commercially valuable  
→ Many products are associated to a color.

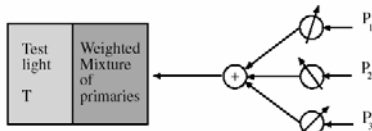
## FILM COLOR MODE



### Controls for

- nearby colors,
- state of mind.

## COLOR MATCHING EXPERIMENTS



Show a split field to subjects

- One side shows the light whose color one wants to measure.
- The other shows a weighted mixture of primaries (fixed lights).  
→ Adjust weights so that both look the same.

## ADDITIVE MATCHING

Many colors can be represented as a mixture of three fixed lights. We write

$$M = aA + bB + cC$$

where the = sign should be read as "matches"

Two people perceive the same color is they agree on a, b, and c.

## SUBTRACTIVE MATCHING

Some colors can't be matched using additive matching. Instead one must write

$$M + a A = b B + c C$$

$$\rightarrow M = b B + c C - a A$$

which can be interpreted as  $(-a, b, c)$ .

Monitor designer's challenge: Choose A, B, C such that positive linear combinations match a large set of colors

## TRICHROMACY

Experimental facts:

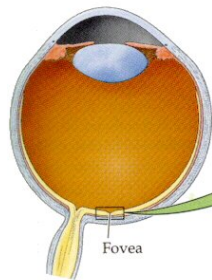
- Three primaries will work for most people if one allows subtractive matching.
  - ❖ Exceptional people can match with two or only one primary.
  - ❖ This could be caused by a variety of deficiencies.
- Most people make the same matches.
  - ❖ There are some anomalous trichromats, who use three primaries but make different combinations to match.

## PERIPHERAL vs FOVEAL VISION

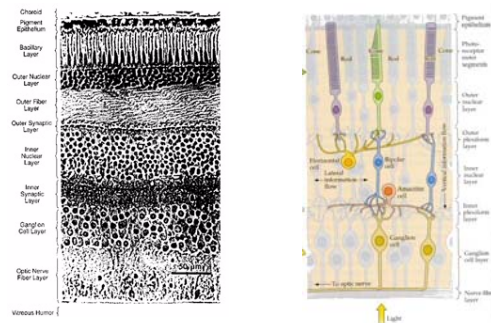
Much higher concentration of cells on the Fovea

→ Active vision:

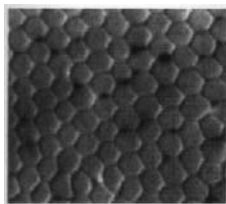
- We find objects using our peripheral vision
- We concentrate our gaze on objects of interest.



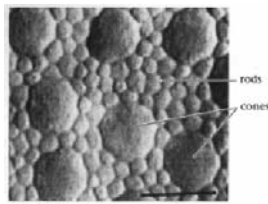
## RETINA



## RETINA

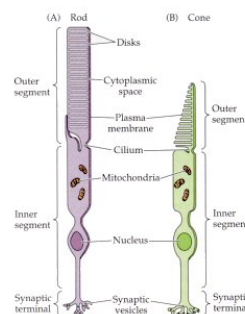


Fovea



Periphery

## RODS AND CONES



Rods: Low-intensity light vision, e.g. night vision

Cones: Color-vision with higher intensity light.

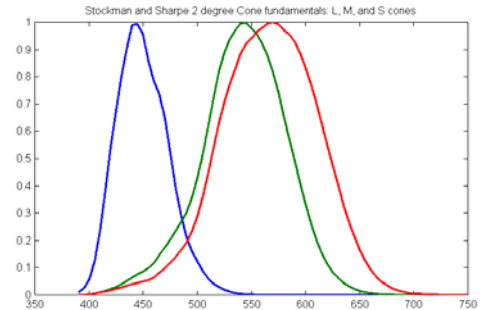
## CONE RESPONSE

Experimental evidence suggests that the response of the k'th type of cone can be written as

$$\int \rho_k(\lambda) E(\lambda) d\lambda$$

where  $\rho_k(\lambda)$  is the sensitivity of the receptor and  $E(\lambda)$  is spectral energy density of the incoming light.

## SENSITIVITY TO DIFFERENT WAVELENGTHS



## GRASSMAN'S LAWS 1853

- Symmetry:  $U=V \iff V=U$
- Transitivity:  $U=V$  and  $V=W \implies U=W$
- Proportionality:  $U=V \iff tU=tV$
- Additivity:  $U=V$  &  $W=X \iff (U+W)=(V+X)$

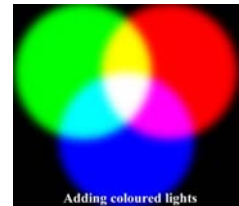
with excellent experimental repeatability.

→ That style of matching is linear in film color mode.

## LINEAR COLOR SPACES

A choice of primaries yields a linear color space:

- The coordinates of a color are given by the weights of the primaries used to match it.
- Choosing the primaries is the same as choosing a color space.



## COLOR MATCHING FUNCTIONS

Given primaries  $P_1, P_2,$  and  $P_3,$  find weights  $f_1(\lambda), f_2(\lambda),$  and  $f_3(\lambda),$  such that

$$U(\lambda) = f_1(\lambda)P_1 + f_2(\lambda)P_2 + f_3(\lambda)P_3,$$

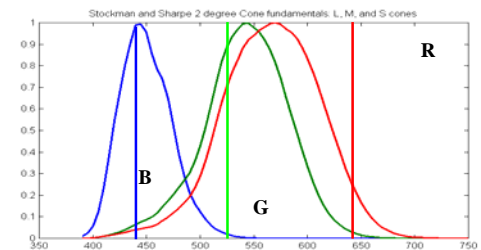
where  $U(\lambda)$  is a unit radiance source at that wavelength.

$$S(\lambda) = w_1P_1 + w_2P_2 + w_3P_3$$

$$= \left\{ \int_{\lambda} f_1(\lambda)S(\lambda)d\lambda \right\} P_1 + \left\{ \int_{\lambda} f_2(\lambda)S(\lambda)d\lambda \right\} P_2 + \left\{ \int_{\lambda} f_3(\lambda)S(\lambda)d\lambda \right\} P_3$$

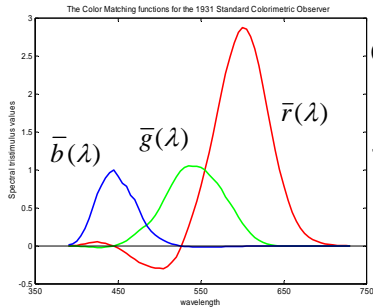
where  $S(\lambda)$  is the sum of sources at many wavelengths.

## RGB COLOR SPACE



Primaries are monochromatic energies around 645.2nm, 526.3nm, 444.4nm.

## RGB MATCHING FUNCTIONS



Color matching functions have negative parts.  
 → Some colors can be matched only subtractively.

## RGB to XYZ

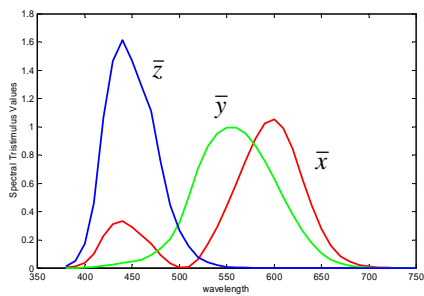
No negative values:

$$\begin{aligned} X &= 0.49R + 0.31G + 0.20B \\ Y &= 0.17697R + 0.81240G + 0.01063B \\ Z &= 0.0R + 0.01G + 0.99B \end{aligned}$$

Equivalent for:

$$\begin{aligned} \bar{x}(\lambda) &= 0.49 \bar{r}(\lambda) + 0.31 \bar{g}(\lambda) + 0.20 \bar{b}(\lambda) \\ \bar{y}(\lambda) &= 0.17697 \bar{r}(\lambda) + 0.81240 \bar{g}(\lambda) + 0.01063 \bar{b}(\lambda) \\ \bar{z}(\lambda) &= 0.0 \bar{r}(\lambda) + 0.01 \bar{g}(\lambda) + 0.99 \bar{b}(\lambda) \end{aligned}$$

## XYZ MATCHING FUNCTIONS

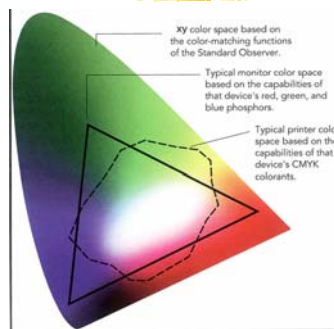


## CHROMATICITY COORDINATES

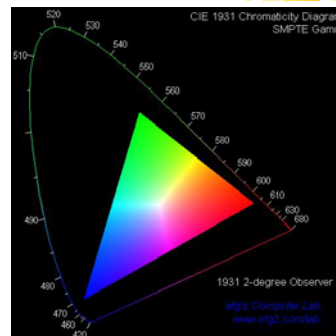
Relative magnitude of tristimulus values:

$$\begin{aligned} x &= X / (X+Y+Z) \\ y &= Y / (X+Y+Z) \\ z &= Z / (X+Y+Z) \\ x + y + z &= 1 \end{aligned}$$

## x,y CHROMATICITY DIAGRAM

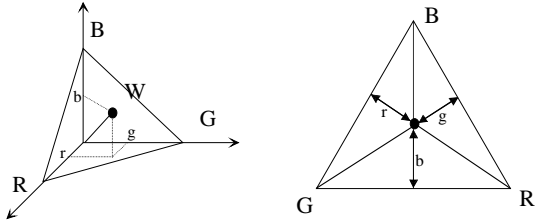


## GAMUT OF A TYPICAL MONITOR



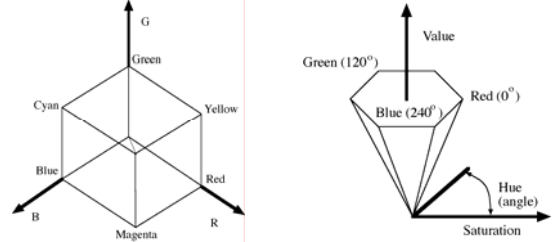
The colors that can be displayed on a typical computer monitor: Phosphor limitations keep the space quite small.

## RGB CHROMATICITY DIAGRAM



The Maxwell triangle involves projecting the colors in RGB space onto  $R+G+B=1$  plane.  
 → Chromaticity independently of luminance.

## RGB HEXCONE



→ Chromaticity expressed in terms of angles and distance from the boundary.

## HSV COLOR SPACE



Hue and saturation are non-linear functions of XYZ because hue relations are naturally expressed on a circle.

## RGB TO HSV

Normalized Colors:

$$r = R / (R + G + B)$$

$$g = G / (R + G + B)$$

$$b = B / (R + G + B)$$

Hue/Saturation/Value:

$$I = R + G + B$$

$$S = 1 - \frac{3 \min(R, G, B)}{I}$$

$$H = \arccos\left(\frac{0.5(2R - G - B)}{\sqrt{(R - G)^2 + (R - B)(G - B)}}\right), \text{ if } B < G$$

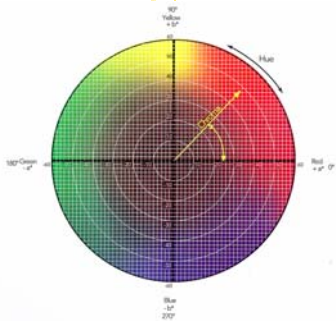
## RGB IMAGES



## HSV IMAGES



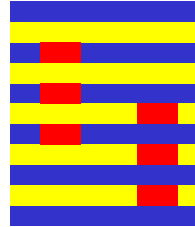
## CIE LAB COLOR SPACE



## CONTEXT

Question: what color is [255,0,0]?

Answer: **red**



How do you capture, encode, and transmit the appearance of this red?

Shevell, 2000

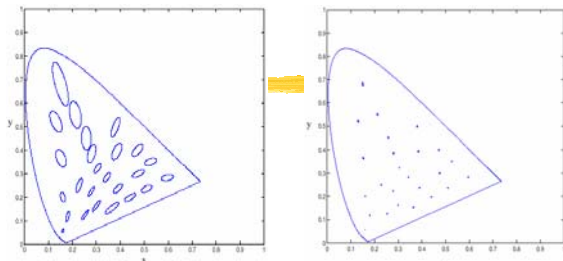
## IN SHORT

- Colors can be expressed in terms of a linear combination of three primaries.
- This is consistent with the fact that there are three kinds of cones in the retina.
- The perception of color is heavily influenced by the context.

## Uniform color spaces

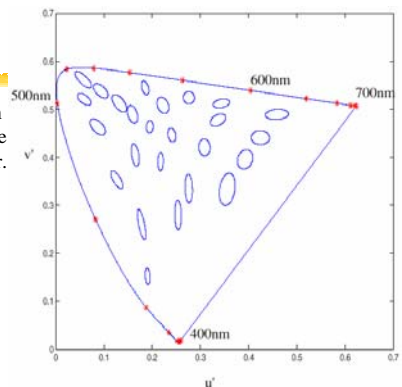
McAdam ellipses (next slide) demonstrate that differences in  $x, y$  are a poor guide to differences in color

Construct color spaces so that differences in coordinates are a good guide to differences in color.



Variations in color matches on a CIE  $x, y$  space. At the center of the ellipse is the color of a test light; the size of the ellipse represents the scatter of lights that the human observers tested would match to the test color; the boundary shows where the just noticeable difference is. The ellipses on the left have been magnified 10x for clarity; on the right they are plotted to scale. The ellipses are known as MacAdam ellipses after their inventor. The ellipses at the top are larger than those at the bottom of the figure, and that they rotate as they move up. This means that the magnitude of the difference in  $x, y$  coordinates is a poor guide to the difference in color.

CIE  $u'v'$  which is a projective transform of  $x, y$ . We transform  $x, y$  so that ellipses are most like one another. Figure shows the transformed ellipses.



## Color receptors and color deficiency

Trichromacy is justified - in color normal people, there are three types of color receptor, called **cones**, which vary in their sensitivity to light at different wavelengths (shown by molecular biologists).

Deficiency can be caused by CNS, by

Some people have fewer than three types of receptor; most common deficiency is red-green color blindness in men.

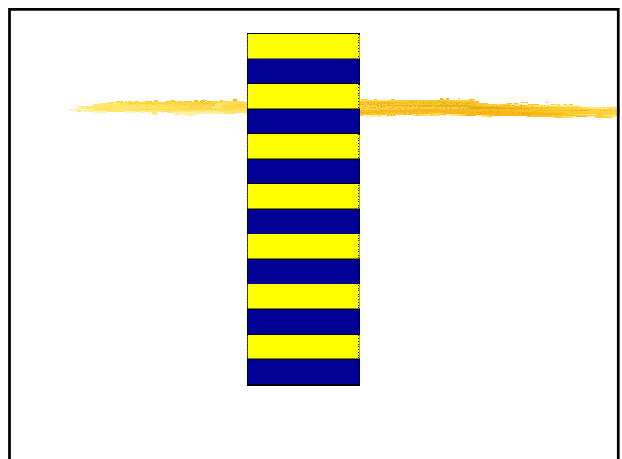
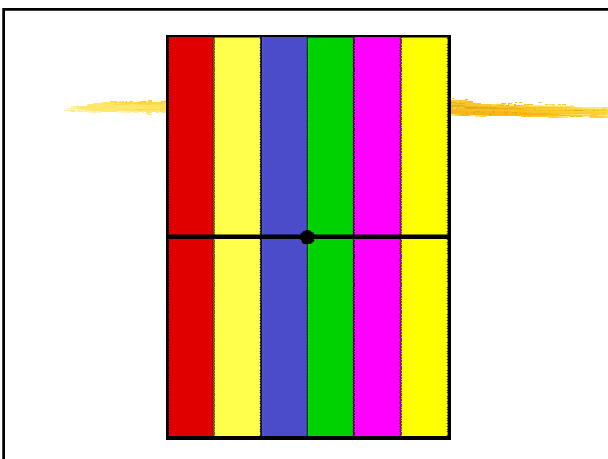
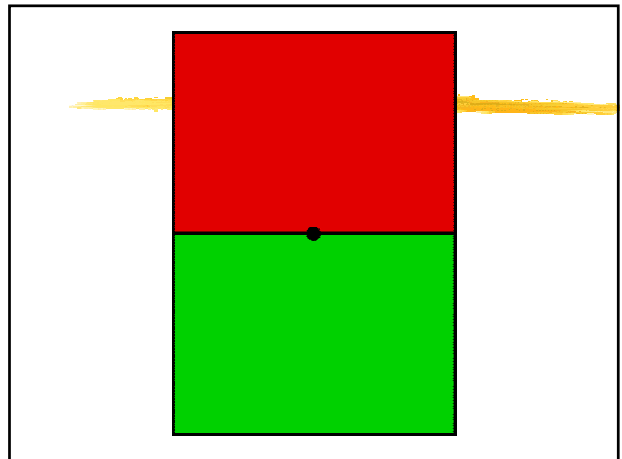
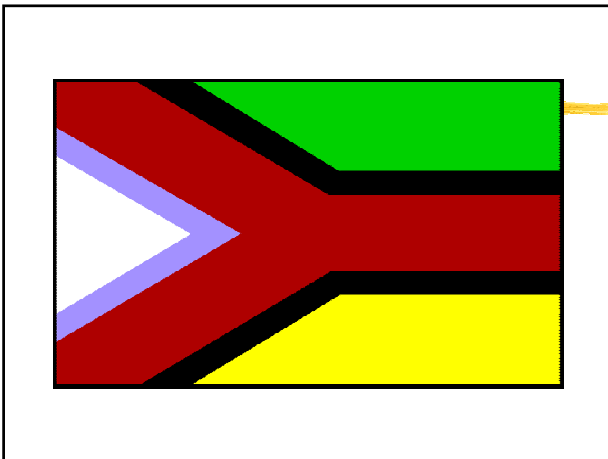
Color deficiency is less common in women; red and green receptor genes are carried on the X chromosome, and

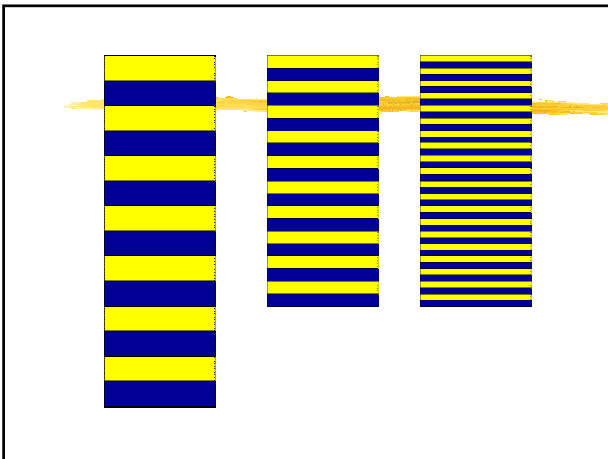
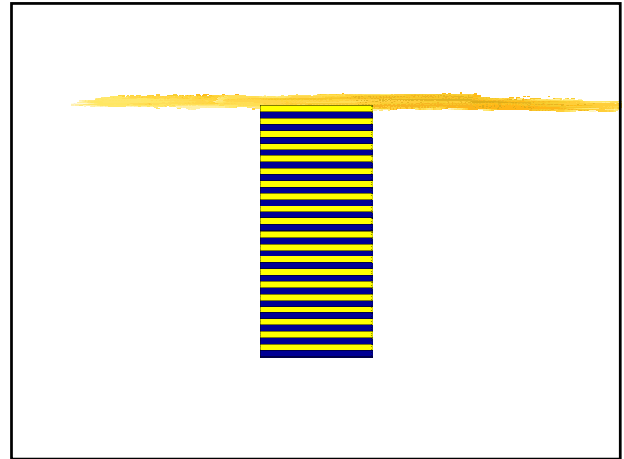
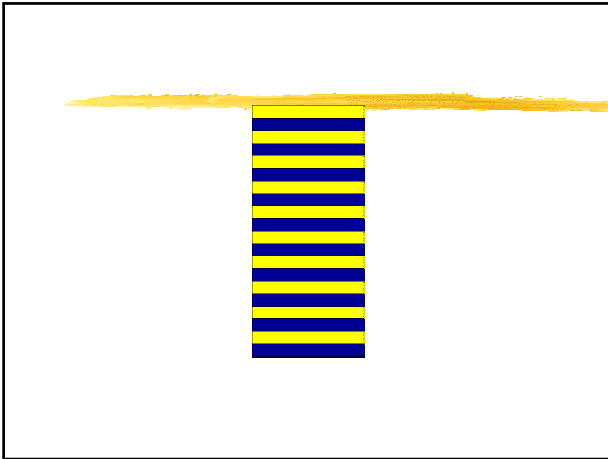
## Adaptation phenomena

The response of your color system depends both on spatial contrast and what it has seen before (adaptation)

This seems to be a result of coding constraints --- receptors appear to have an operating point that varies slowly over time, and

Common example: walk inside from a bright day; everything looks dark for a bit, then takes its conventional brightness.





## Viewing coloured objects

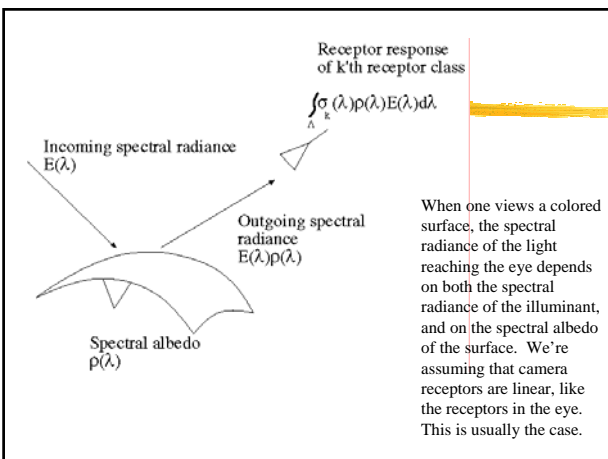
Assume diffuse+specular model

Specular

- specularities on dielectric objects take the colour of the light
- specularities on metals can be coloured

Diffuse

- colour of reflected light depends on both illuminant and surface
- people are surprisingly good at disentangling these effects in practice (colour constancy)
- this is probably where some of the spatial phenomena in colour perception come from



## Subtractive mixing of inks

Inks subtract light from white, whereas phosphors glow.

Linearity depends on pigment properties

- inks, paints, often hugely non-linear.

Inks: Cyan=White-Green, Magenta=White-Red, Yellow=White-Blue.

For a good choice of inks, **and good**

eg. C+M+Y=White-White=Black  
C+M=White-Yellow=Blue

Usually require CMY and Black, because colored inks are more expensive, and registration is hard

For good choice of inks, there is a linear transform between XYZ and CMY

## Finding Specularities

Assume we are dealing with dielectrics

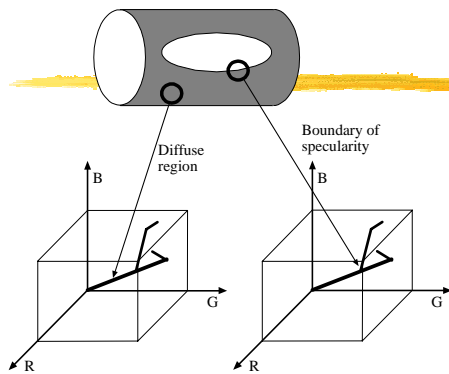
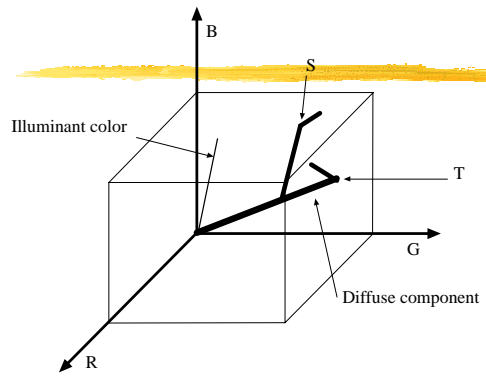
- specularly reflected light is the same color as the source

Reflected light has two components

- diffuse
- specular
- and we see a weighted sum of these two

Specularities produce a characteristic dogleg in the histogram of receptor responses

*in a patch of diffuse surface, we see a color*



## Color constancy

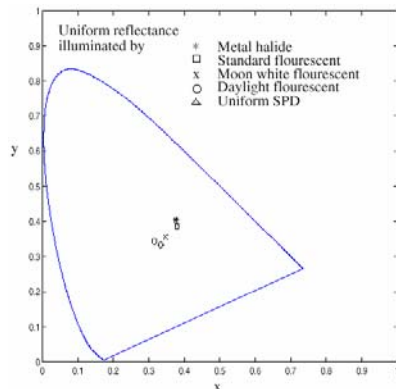
Assume we've identified and removed specularities

The spectral radiance at the camera depends on two things

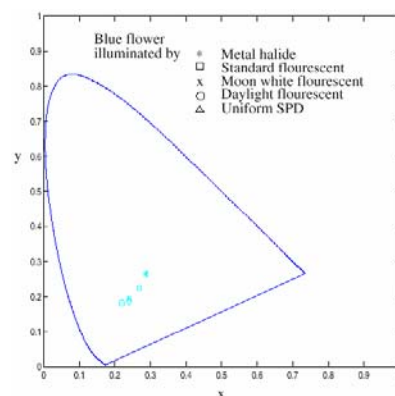
- surface albedo
- illuminant spectral radiance
- the effect is much more pronounced than most people think (see following slides)

We would like an illuminant invariant description of the surface

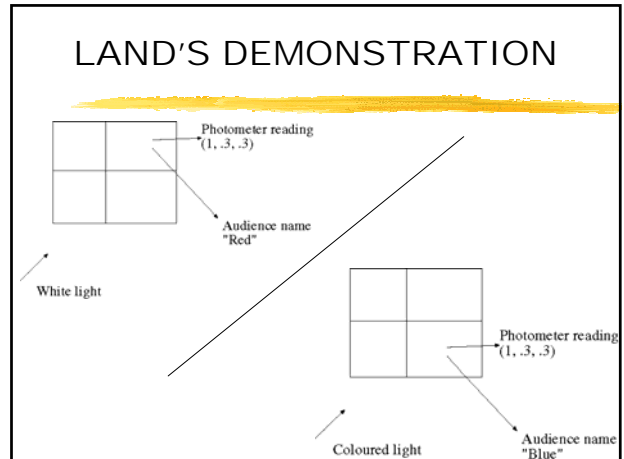
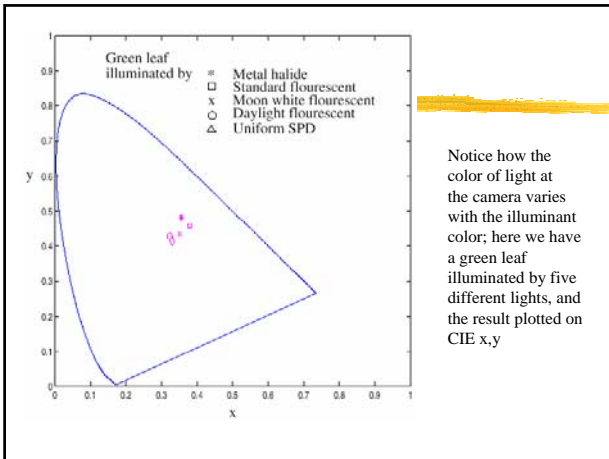
- e.g. some measurements of surface albedo
- need a model of the interactions



Notice how the color of light at the camera varies with the illuminant color; here we have a uniform reflectance illuminated by five different lights, and the result plotted on CIE x,y



Notice how the color of light at the camera varies with the illuminant color; here we have the blue flower illuminated by five different lights, and the result plotted on CIE x,y. Notice how it looks significantly more saturated under some lights.



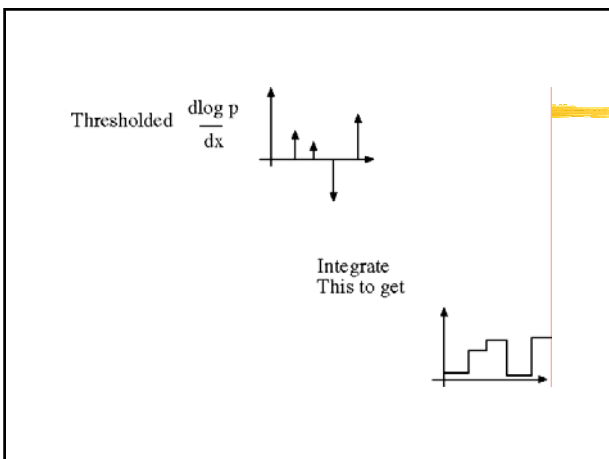
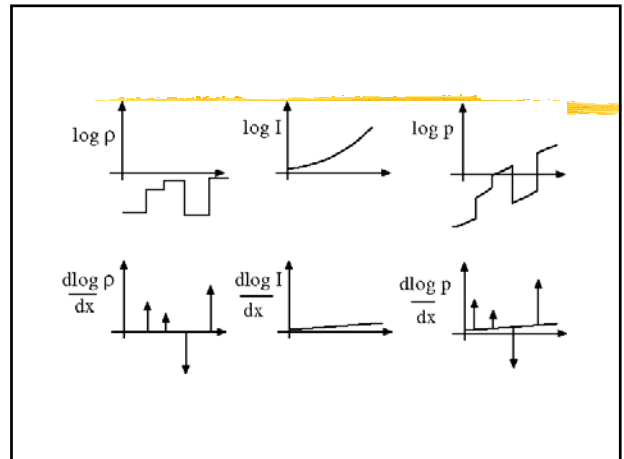
## Lightness Constancy

Lightness constancy

- how light is the surface, independent of the brightness of the illuminant
- issues
  - spatial variation in illumination
  - absolute standard
- Human lightness constancy is very good

Assume

- frontal 1D "Surface"
- slowly varying illumination
- quickly varying surface reflectance



## Lightness Constancy in 2D

Differentiation, thresholding are easy

- integration isn't
- problem - gradient field may no longer be a gradient field

This yields a minimization problem

How do we choose the constant of integration?

- average lightness is grey
- lightest object is white
- ?

One solution

- Choose the function whose gradient is "most like" thresholded gradient

## Simplest colour constancy

Adjust three receptor channels independently

- Von Kries
- Where does the constant come from?
  - White patch
  - Averages
  - Some other known reference (faces, nose)

## Colour Constancy - I

We need a model of interaction between illumination and surface colour

- finite dimensional linear model seems OK

Finite Dimensional Linear Model (or FDLM)

- surface spectral albedo is a weighted sum of basis functions
- illuminant spectral exitance is a weighted sum of basis functions
- This gives a quite simple form to interaction between the two

## Finite Dimensional Linear Models

Receptor response of k'th receptor class

$$p_k = \int \sigma_k(\lambda) \rho(\lambda) E(\lambda) d\lambda$$

$$E(\lambda) = \sum_{i=1}^m \varepsilon_i \psi_i(\lambda)$$

$$\rho(\lambda) = \sum_{j=1}^n r_j \varphi_j(\lambda)$$

$$p_k = \int \sigma_k(\lambda) \left( \sum_{i=1}^m \varepsilon_i \psi_i(\lambda) \right) \left( \sum_{j=1}^n r_j \varphi_j(\lambda) \right) d\lambda$$

$$= \sum_{i=1, j=1}^{m, n} \varepsilon_i r_j \int \sigma_k(\lambda) \psi_i(\lambda) \varphi_j(\lambda) d\lambda$$

$$= \sum_{i=1, j=1}^{m, n} \varepsilon_i r_j g_{ijk}$$

## General strategies

Determine what image would look like under white light

Assume

- that we are dealing with flat frontal surfaces
- We've identified and removed specularities
- no variation in illumination

We need some form of reference

- brightest patch is white
- spatial average is known
- gamut is known
- specularities

## Obtaining the illuminant from specularities

Assume that a specularity has been identified, and material is dielectric.

Then in the specularities we have

$$= \sum_{i=1}^m \varepsilon_i \int \sigma_k(\lambda) \psi_i(\lambda) d\lambda$$

Assuming

- we know the sensitivities and the illuminant basis functions
- there are no more illuminant basis functions than receptors

This linear system yields the illuminant coefficients.

## Obtaining the illuminant from average color assumptions

Assume the spatial average reflectance is known

$$\rho(\lambda) = \sum_{j=1}^n r_j \varphi_j(\lambda)$$

Assuming

- $g_{ijk}$  are known
- average reflectance is known
- there are not more receptor types than illuminant basis functions

We can measure the spatial average of the receptor response to get

We can recover the illuminant coefficients from this linear system

## Computing surface properties

Two strategies

- compute reflectance coefficients
- compute appearance under white light.

These are essentially equivalent.

Once illuminant coefficients are known, to get reflectance coefficients we solve the linear system

to get appearance under white light, plug in reflectance coefficients and compute  $\sum_{i=1}^{m_i} \epsilon_i^{white} r_j g_{ijk}$