IMAGE FORMATION

From 3-D physical world to 2-D image:

- Analog vs digital images
- Camera geometry
- Radiometry and sensing

ANALOG IMAGES

An image on film can be understood a two-dimensional light intensity function f(x,y) where:

- x and y are spatial coordinates
- The value of f at any point (x, y) is proportional to the brightness or gray value of the image at that point.

→ Cannot be stored as such on a digital computer.

DIGITAL IMAGES

A digitized image is one in which:

- Spatial and grayscale values have been made discrete.
- Intensities measured across a regularly spaced grid in x and y directions are sampled to
  - 8 bits (256 values) per point for black and white,
  - 3x8 bits per point for color images.

They are stored as a two dimensional arrays of gray-level values. The array elements are called pixels and identified by their x, y coordinates.

PIXELS

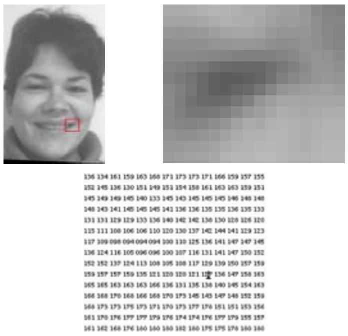
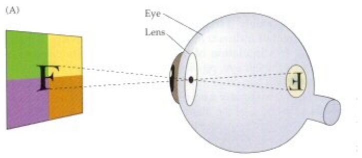


IMAGE FORMATION

1. Light from world surfaces into the lens of the camera.
2. Photons hit CCD array that produce electric charges.
3. Charges read as voltages.
4. Voltages sampled by a digitizer (or frame grabber)

→ Array of pixel values

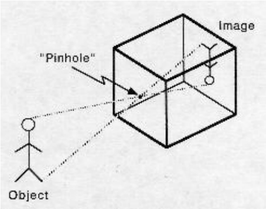
CAMERAS



Projection from surfaces to 2-D sensor.

- Where: Geometry
- How bright: Radiometry
- Stored how: Sensing

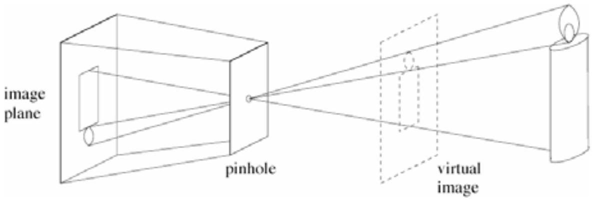
PINHOLE CAMERA MODEL



Idealized model that defines perspective projection:

- All rays go through a hole and form a star of lines
- The hole acts as a selector of rays that allows the formation of an inverted image.

VIRTUAL IMAGE



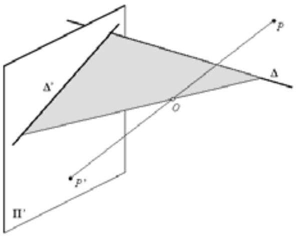
GEOMETRIC PROPERTIES

In General

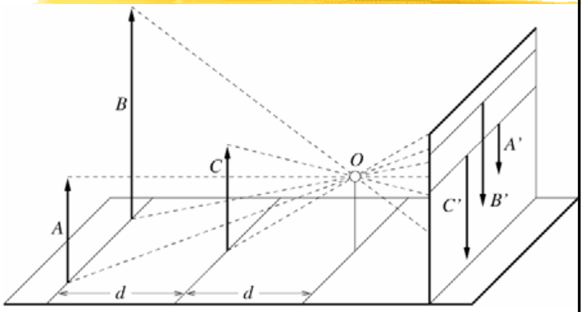
- Points go to points
- Lines go to lines
- Planes go to whole image
- Polygons go to polygons

Degenerate cases

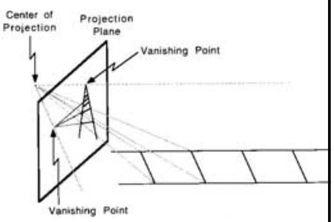
- Lines through focal point go to point
- Planes through focal point go to line



DISTANT OBJECTS APPEAR SMALLER

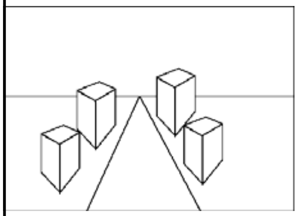


PARALLEL LINES MEET



Parallel lines in 3-D project to a pencil of 2-D lines that meet at one point.

VANISHING POINTS

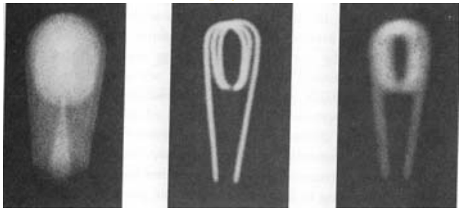


- Each set of parallel lines meets at a different point
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.

→Good ways to spot faked images

- Scale and perspective don't match
- Vanishing points behave badly

LIMITATIONS

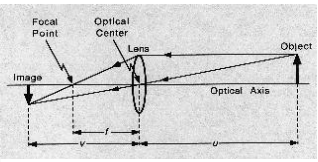


Idealization because the hole cannot be infinitely small

- Image would be infinitely dim
- Diffraction effects

→ Use of Lenses.

THIN LENSES

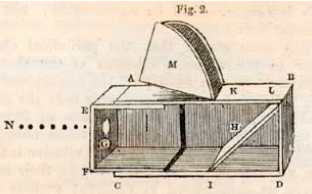


Thin lens Equation:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

→ Lens with focal distance f equivalent to pinhole camera with similar focal distance but larger aperture.

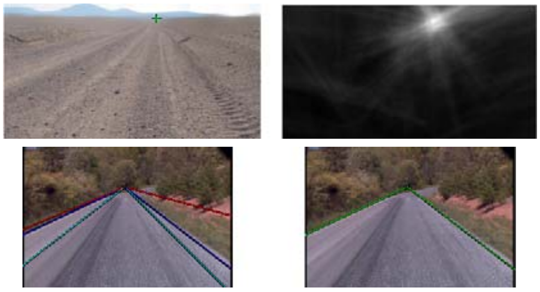
CAMERA OBSCURA



- Used by painters since the Renaissance to produce perspective projections.
- Direct ancestors to the first film cameras.

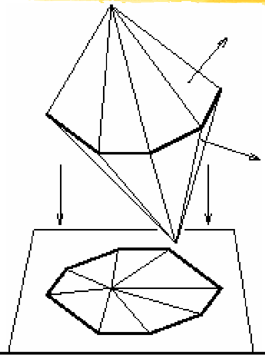
Art

ROAD DETECTION



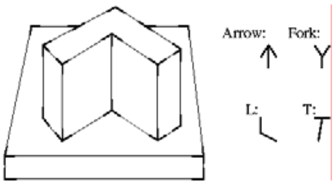
C. Rasmussen, Texture-Based Vanishing Point Voting for Road Shape Estimation,In *British Machine Vision Conference,* Kingston, UK, 2004.

POLYHEDRA PROJECT TO POLYGONS



Because lines project to lines

JUNCTIONS ARE CONSTRAINED



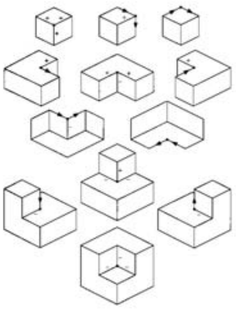
In theory such a drawing, can be interpreted as the projection of a 3—D object:

- In theory, one looks for consistent sets of labels.
- In practice, hard to get the lines and junctions to label from real images

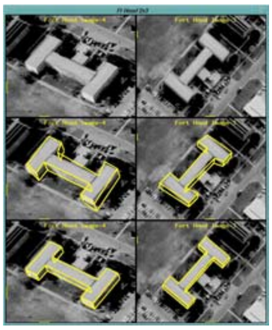
THE BLOCKS WORLD

- Computer Vision started in 1965 at MIT as a short term project.
- Perfect data and strong assumptions.

→ The real world is not like that!

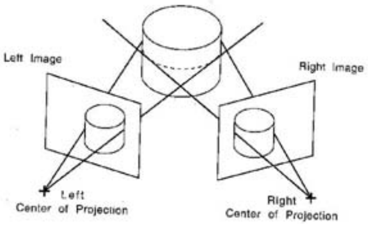


DEFORMABLE MODELS



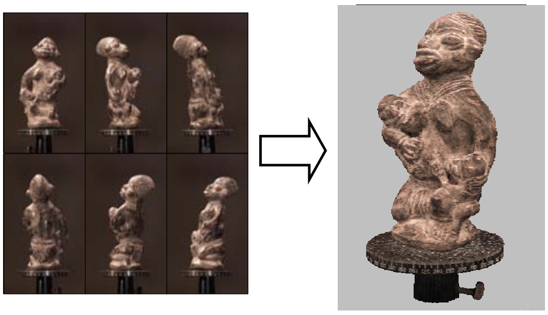
The deformable model encodes the endpoints of the segments.

CURVED SURFACES PRODUCE OCCLUDING CONTOURS



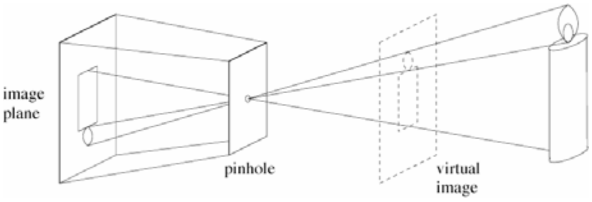
- Occluding contour is the set of points where the viewing direction is tangent to the surface
- This is a projection of a space curve, which varies from view to view of the surface

MODELING FROM MANY PHOTOGRAPHS

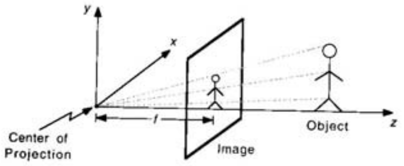


C. Hernandez, PhD ENST, 2004

VIRTUAL IMAGE



PERSPECTIVE PROJECTION

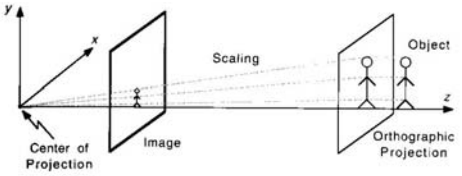


$$u = f\,\frac{x}{z}$$

$$v = f\,\frac{y}{z}$$

Pinhole geometry without image reversal

ORTHOGRAPHIC PROJECTION



$$u = sx$$
$$v = sy$$

Special case of perspective projection:

- Large f
- Objects close to the optical axis

→Parallel lines mapped into parallel lines.

PROJECTIVE DISTORSIONS

Orthographic and parallel to image plane.

Orthographic but not parallel to image plane.

Full perspective.

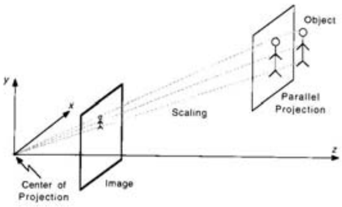
PROJECTIVE DISTORTIONS

Perspective

Orthographic

SHAPE FROM TEXTURE

PARAPESPECTIVE PROJECTION

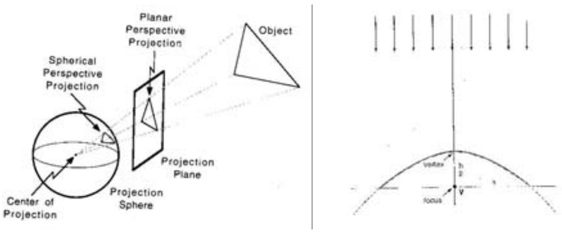


Generalization of the orthographic projection:

- Object dimensions small wrt distance to the center of projection.

→Parallel Projection followed by scaling

SPHERICAL PROJECTION



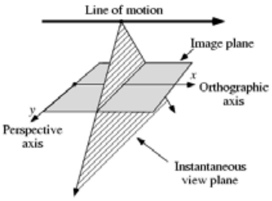
OMNI DIRECTIONAL CAMERA

SPHERICAL TO PROJECTIVE

MOBILE ROBOTS

Clodbuster, Grasp Lab, U. Penn.

PUSHBROOM CAMERAS



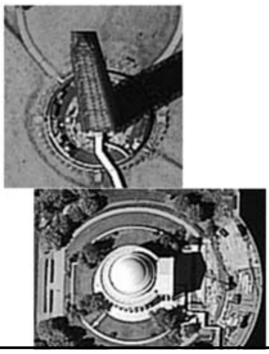
$$u = sx$$

$$v = f\frac{y}{z}$$

-> Straight lines project to hyperbolas

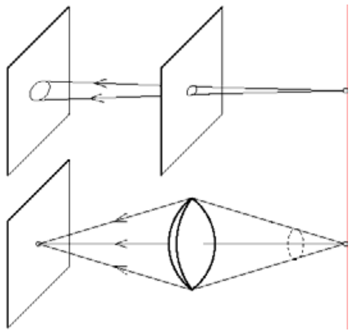
IKONOS
1999

WASHINGTON MONUMENTS SEEN FROM SPACE



- Altitude 700 km
- Ground sampling distance < 1m

IMAGING WITH A LENS



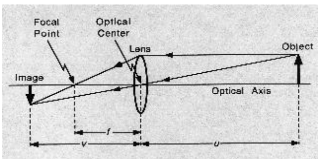
FROM BLUR TO BLUR

Pinhole too big:
Many directions are averaged, blurring the image

Pinhole too small:
Diffraction effects blur the image

In general, pinhole-camera images are *dark* because only a very small set of rays from any particular point hits the screen.

THIN LENSES

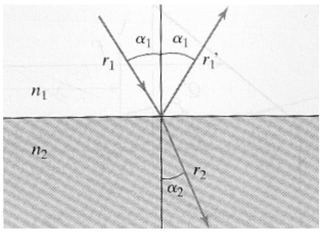


Thin lens Equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

→ Lens with focal distance f equivalent to pinhole camera with similar focal distance but larger aperture.

REFRACTION



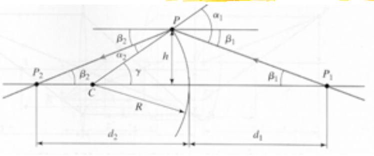
Snell's law:

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

where n_1 and n_2 are the indices of refraction.

Forsyth & Ponce, Chap 1.2

PARAXIAL OPTICS



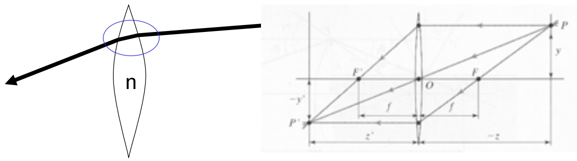
$$\alpha_1 = \gamma + \beta_1 \approx h\left(\frac{1}{R} + \frac{1}{d_1}\right) \text{ and } \alpha_2 - \gamma + \beta_2 \approx h\left(\frac{1}{R} + \frac{1}{d_2}\right)$$

Therefore

$$n_1\alpha_1 \approx n_2\alpha_2 \Rightarrow \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$$

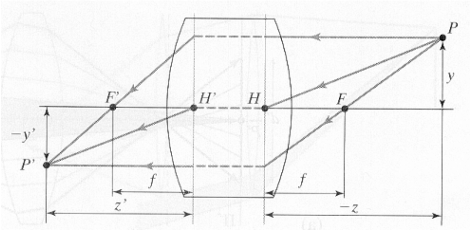
- d_1 and d_2 do not depend on β_1 and β_2.
- still holds if some of the values of d_1, d_2 and R change sign.

THIN LENS PROPERTIES



- Two spherical surfaces form a thin lens if a ray entering refracted at its right boundary if immediately refracted at its left boundary.
- Rays going through the optical center are not refracted.
- Its focal length is $f = \frac{R}{2(n-1)}$

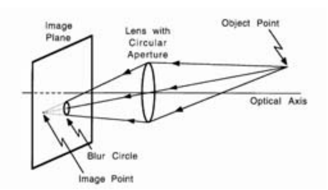
THICK LENSES



Same as thin lenses except for an offset

→ The only undeflected ray is along the optical axis

DEPTH OF FIELD vs APERTURE



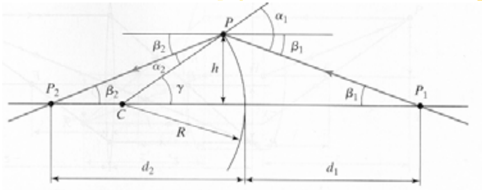
Large Aperture:
- Large blur circles
- Shallow depth of field

Small Aperture:
- Low intensity
- Long exposure time

DISTORTIONS

The lens is not exactly a “thin lens:”
- Different wave lengths refracted differently
- Barrel Distortion

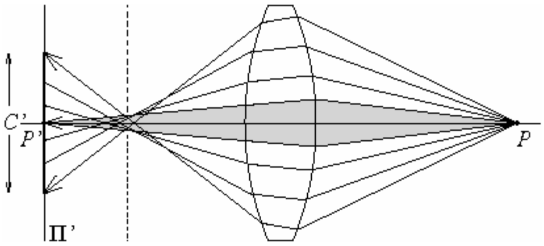
PARAXIAL OPTICS REVISITED



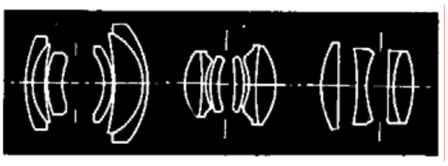
$$\frac{n_1}{d_1}+\frac{n_2}{d_2}=\frac{n_2-n_1}{R}+h^2\left[\frac{n_1}{2d_1}\left(\frac{1}{R}+\frac{1}{d_1}\right)^2+\frac{n_2}{2d_2}\left(\frac{1}{R}-\frac{1}{d_2}\right)^2\right]$$

→ Distortion increases with distance away from the optical axis.

SPHERICAL ABERRATION



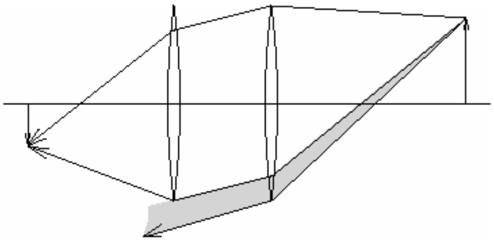
LENS SYSTEMS



Aberrations can be minimized by aligning several lenses with well chosen

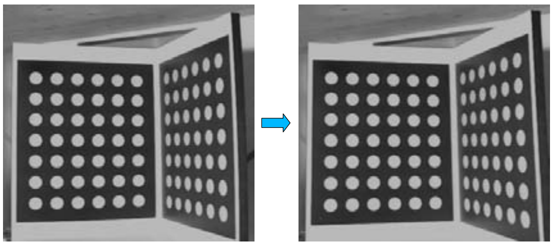
- Shapes,
- Refraction indices.

VIGNETTING

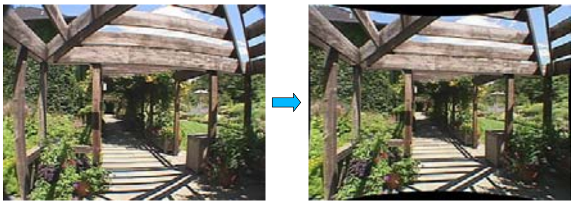


Images can get darker towards their edges because some of the light does not go through all the lenses.

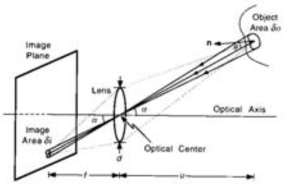
UNDISTORTING



UNDISTORTING



RADIOMETRY



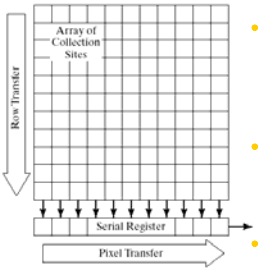
Scene Radiance: Amount of light radiation from a surface point (Watt / m2 / Steradian)

Image Irradiance: Amount of light incident at the image of the surface point. (Watt / m2)

Fundamental Radiometric Equation:

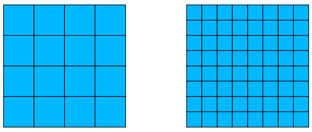
$$Irr = \frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha) Rad$$

CHARGE COUPLED DEVICE



- Photons free up electrons that are then captured by a potential well.
- Charges are transferred row by row wise to a register.
- Pixel values are read from the register.

SENSING



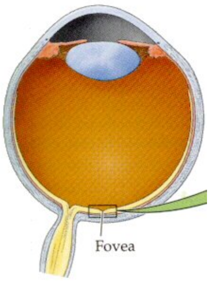
Conversion of the “optical image” into an “electrical image”:

$$E(x,y) = \iint \mathrm{Irrad}(x,y,t,\lambda) s(\lambda) \tau(t - t_0) d\lambda dt$$

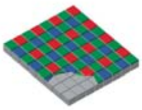
$$I(m,n) = \mathrm{Quantize}(\iint E(x,y) \omega(x-m, y-n) dxdy)$$

→ Quantization in
- Time
- Space

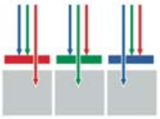
HUMAN EYE



MOSAIC CAPTURE



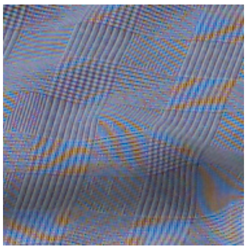
In conventional systems, color filters are applied to a single layer of pixels in a tiled mosaic pattern.



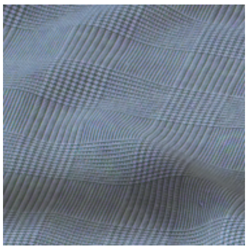
The filters let only one wavelength of light–red, green, or blue–pass through to any given pixel, allowing it to record only one color.

As a result, mosaic sensors capture only 25% of the red and blue light, and just 50% of the green.

COLOR ARTIFACTS

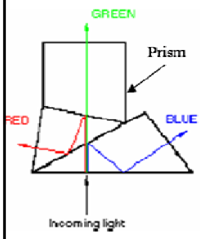


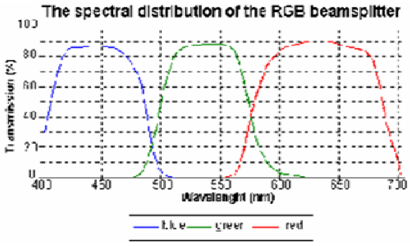
Color interpolation creates artifacts.



Artifacts go away when using three CCDs.

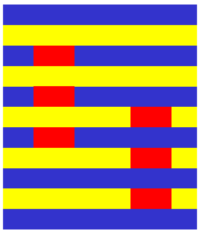
BEAM SPLITTER





COLOR

Question: what color is [255,0,0]?
Answer: red



How do you capture, encode, and
transmit the appearance of this red?

Shevell, 2000

COLORED SPIRALS