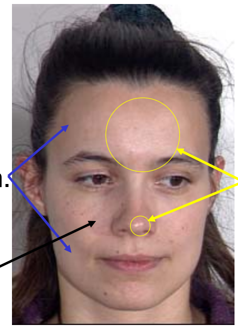
SOURCES, SHADOWS AND SHADING

Illumination models:
- Radiometry
- Point light sources
- Area light sources
- Local vs global models

DIRECT LIGHTING

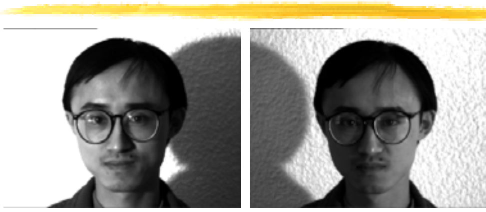
Diffuse reflection.



Specularities

Shadow.

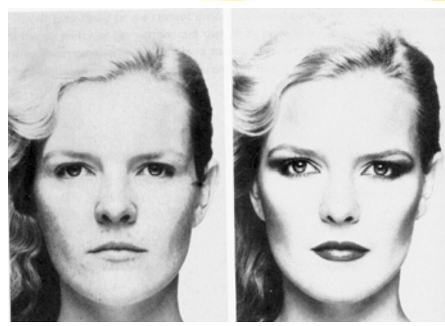
EFFECT OF ILLUMINATION



Light source strength and direction has a dramatic impact on distribution of brightness in the image:
- Shadows
- Highlights
- ...

Yale face database

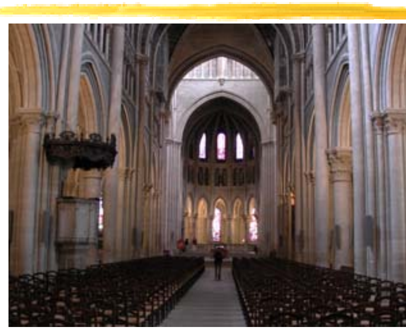
SHAPE FROM SHADING



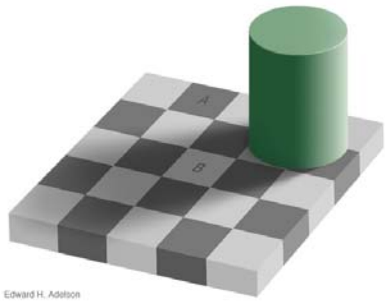
TEAPOTS

The original and some of its many copies.

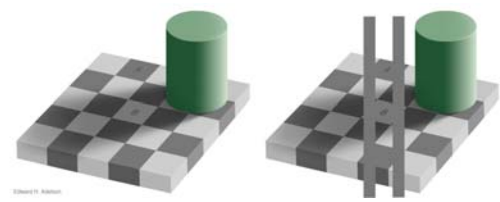
INDIRECT LIGHTING



ILLUSION



NO MORE ILLUSION



The human eye measures relative rather than absolute intensity values.

SHADING MODELS

Local shading model
- Surface has radiosity due only to sources visible at each point.

Advantages:
- Easy to express/manipulate
- Supports relatively simple theories of how shape information can be extracted from shading

Disadvantage:
- Very inaccurate

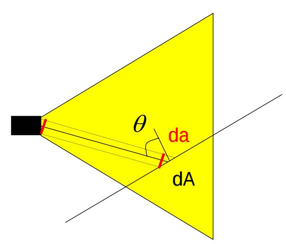
Global shading model
- Surface radiosity is due to radiance reflected from other surfaces as well.

Advantages:
- Usually very accurate

Disadvantage:
- Extremely difficult to infer anything from shading values

Forsyth & Ponce, Chap 4 Art

LOCAL SHADING MODEL

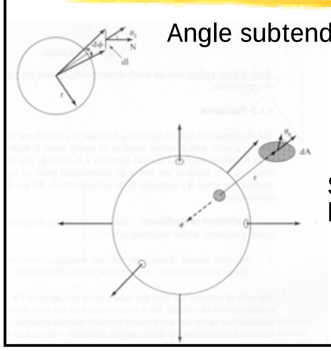


Foreshortening:

$$da = \cos(\theta) dA$$

The effect of a distant source on a surface patch depends on its **apparent** surface.

FROM ANGLE TO SOLID ANGLE



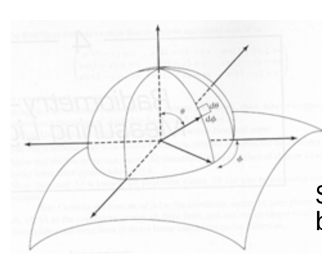
Angle subtended by segment l:

$$d\phi = \frac{dl \cos(\theta_l)}{r}$$

Solid angle subtended by surface dA:

$$d\omega = \frac{dA \cos(\theta_n)}{r^2}$$

SPHERICAL COORDINATES



Orientations can be described in terms of two angles:

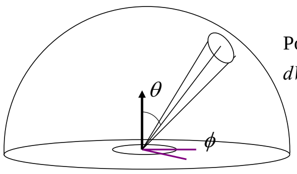
θ and ϕ

Solid angle subtended by surface $d\phi$ $d\theta$:

$$d\omega = \sin(\theta) d\theta d\varphi$$

RADIANCE

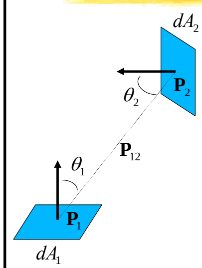
Amount of light radiation from a surface point per unit area perpendicular to the direction of travel (Watt/m2 /Steradian)



Power emitted in direction (θ,ϕ):

$dW = L(\mathbf{P},\theta,\phi)(\cos(\theta)dA)d\omega$

RADIANCE



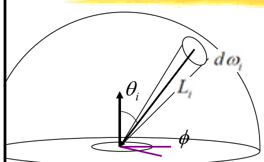
$$dW_{1\to 2} = L(\mathbf{P}_1,\mathbf{P}_1\mathbf{P}_2)(\cos(\theta_1)dA_1)d\omega_2$$
$$= L(\mathbf{P}_1,\mathbf{P}_1\mathbf{P}_2)(\cos(\theta_1)dA_1)(\frac{\cos(\theta_2)dA_2}{r^2})$$
$$dW_{2\to 1} = L(\mathbf{P}_1,\mathbf{P}_2\mathbf{P}_1)(\cos(\theta_2)dA_2)d\omega_1$$
$$= L(\mathbf{P}_1,\mathbf{P}_2\mathbf{P}_1)(\cos(\theta_2)dA_2)(\frac{\cos(\theta_1)dA_1}{r^2})$$

Therefore:

$$dW_{1\to 2} = dW_{2\to 1} \Rightarrow L(\mathbf{P}_1,\mathbf{P}_1\mathbf{P}_2) = L(\mathbf{P}_1,\mathbf{P}_2\mathbf{P}_1)$$

→ Radiance is constant along a straight line.

IRRADIANCE



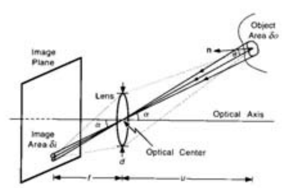
Amount of light incident per unit of unforeshortened area. (Watt / m2)

Patch of surface dA illuminated by radiance $L_i(P,\theta_i,\phi_i)$ coming from solid angle $d\omega$ receives irradiance

$$(1/dA)L_i(P,\theta_i,\phi_i)(\cos(\theta_i)dA)d\omega = L_i(P,\theta_i,\phi_i)\cos(\theta_i)d\omega$$

→ Irradiance is radiance × foreshortening × solid angle.

RADIOMETRY



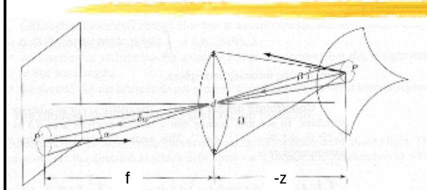
Scene Radiance: Amount of light radiation from a surface point (Watt / m2 / Steradian)

Image Irradiance: Amount of light incident at the image of the surface point. (Watt / m2)

Fundamental Radiometric Equation:

$$Irr = \frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4(\alpha) Rad$$

RADIOMETRIC EQUATION (1)



$\delta P = L\,\delta A\cos(\beta)\,\Omega$ power emitted from δA and falling on the lens, where L is the radiance

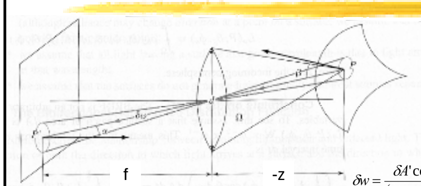
$$\Omega = \frac{\pi}{4}\frac{d^2\cos(\alpha)}{(z/\cos(\alpha))^2} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2\cos^3(\alpha)$$

therefore

$$I = \frac{\delta P}{\delta A'} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 L\frac{\delta A}{\delta A'}\cos^3(\alpha)\cos(\beta)$$

Forsyth & Ponce, Chap 4.1

RADIOMETRIC EQUATION (2)



$$\delta w = \frac{\delta A'\cos(\alpha)}{(f/\cos(\alpha))^2} = \frac{\delta A\cos(\beta)}{(z/\cos(\alpha))^2}$$

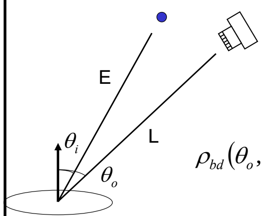
$$\Rightarrow \frac{\delta A}{\delta A'} = \frac{\cos(\alpha)}{\cos(\beta)}\left(\frac{z}{f}\right)^2$$

Therefore $I = \frac{\delta P}{\delta A'} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 L\frac{\delta A}{\delta A'}\cos^3(\alpha)\cos(\beta)$

$$= \left[\frac{\pi}{4}\left(\frac{d}{f}\right)^2\cos^4(\alpha)\right]L$$

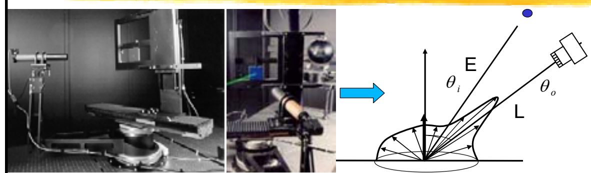
BRDF

Bidirectional Reflectance Distribution Function:
Ratio of the radiance in the outgoing direction to the incident irradiance.



$$\rho_{bd}(\theta_o,\phi_o,\theta_i,\phi_i)=\frac{L_o(P,\theta_o,\phi_o)}{L_i(P,\theta_i,\phi_i)\cos(\theta_i)d\omega}$$

MEASURING THE BRDF

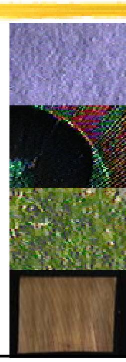


Gonioreflectometer:

- Shine real light off a real material.
- Measure reflected light.

COLUMBIA-UTRECHT REFLECTANCE AND TEXTURE DATABASE

http://www1.cs.columbia.edu/CAVE/curet/



BRDF MEASUREMENTS

	θ_o	ϕ_o	θ_i	ϕ_i	Measurement
1	1.389566	-1.970402	1.374447	-1.570796	x_1
2	0.981748	0.000000	1.374447	0.000000	x_2
3	1.463021	-2.763231	1.370707	-2.376765	x_3
⋮	⋮	⋮	⋮	⋮	
205	1.464020	-3.065729	1.357866	-0.153484	x_{205}

- Hard to repeat exactly
- Depends on the color of light source

ENERGY CONSERVATION

Outgoing radiance:

$$L_o(P,\theta_o,\phi_o)=\int_{\Omega_i}\rho_{bd}(\theta_o,\phi_o,\theta_i,\phi_i)L_i(P,\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

$$L_o(P)=\int_{\Omega_o}L_o(P,\theta_o,\phi_o)\cos(\theta_o)d\omega_o$$

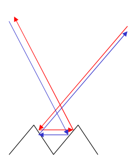
Incoming radiance:

$$L_i(P)=\int_{\Omega_i}L_i(P,\theta_i,\phi_i)\cos(\theta_i)d\omega_i$$

Energy Conservation:

$$\int_{\Omega_i}L_i(P,\theta_i,\phi_i)\cos(\theta_i)d\omega_i\geq\int_{\Omega_i}\int_{\Omega_o}\rho_{bd}(\theta_o,\phi_o,\theta_i,\phi_i)L_i(P,\theta_i,\phi_i)\cos(\theta_i)\cos(\theta_o)d\omega_i d\omega_o$$

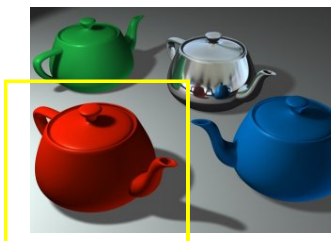
HELMHOLTZ RECIPROCITY



- Incoming to outgoing pathway same as outgoing to incoming pathway
- The BRDF is symmetric in the incoming and outgoing directions.

$$\rho_{bd}(\theta_o,\phi_o,\theta_i,\phi_i)=\rho_{bd}(\theta_i,\phi_i,\theta_o,\phi_o)$$

MATTE SURFACES



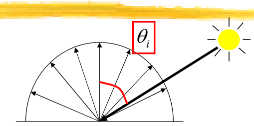
Radiance depends on the incoming angle but not the outgoing one.

DIFFUSE REFLECTION

Constant BRDF:

$$\rho = \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i)$$

Outgoing radiance:



$$\begin{aligned}\rho_d &= \int_{\Omega_o} \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i)\cos(\theta_o)d\omega_o \\ &= \rho\int_{\Omega_o} \cos(\theta_o)d\omega_o \\ &= \rho\int_{\Omega_o} \cos(\theta_o)\sin(\theta_o)d\theta_o d\phi_o \\ &= \rho\pi\end{aligned}$$

$$\Rightarrow \rho = \frac{\rho_d}{\pi} \text{ with } 0 \le \rho_d \le 1$$

RADIOSITY

Surface patch with constant radiance:

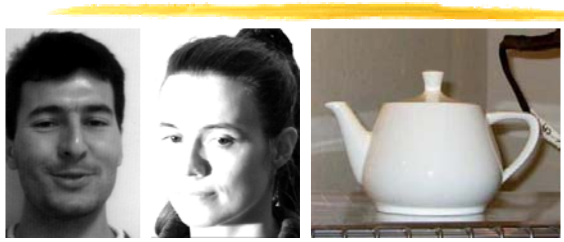
$$\begin{aligned}B(P) &= \int_\Omega L_0(P)\cos(\theta)d\omega \\ &= L_0(P)\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\cos(\theta)\sin(\theta)d\omega \\ &= \pi L_0(P)\end{aligned}$$

Radiosity is the total energy leaving the surface in all directions.

SPECULAR SURFACES

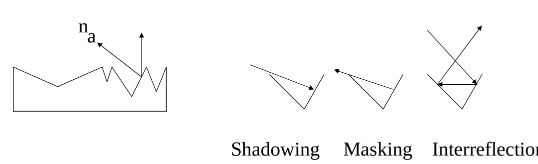
Perfect mirror reflects light in only one specific direction. Radiance now depends on both the incoming angle and the outgoing one.

REAL SURFACES



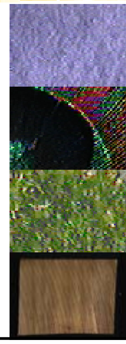
→ Mixture of specular and Lambertian.

MICROSTRUCTURES



- Smooth surface → Specular reflection
- Rough surface → Diffuse reflection

COLUMBIA-UTRECHT REFLECTANCE AND TEXTURE DATABASE



Felt

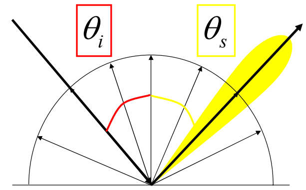
Peacock Feather

Moss

Wood

http://www1.cs.columbia.edu/CAVE/curet/

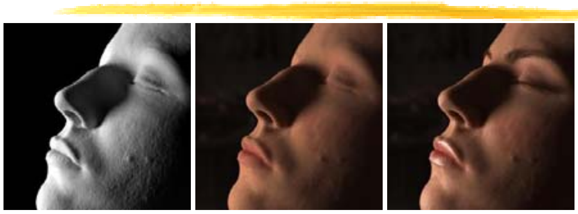
SPECULAR REFLECTION



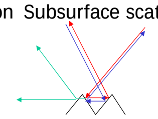
Radiance is the sum of a diffuse and a specular component:

$$L(P,\theta_0,\phi_0) = \rho_d(P)\int_\Omega L(P,\theta_i,\phi_i)\cos(\theta_i)d\omega + \rho_s(P)L(P,\theta_s,\phi_s)\cos^n(\theta_s-\theta_0)$$

SUBSURFACE SCATTERING



Diffuse reflection Subsurface scattering Complete rendering



H. Wann Jensen SIGGRAPH'03

LIGHT SOURCES

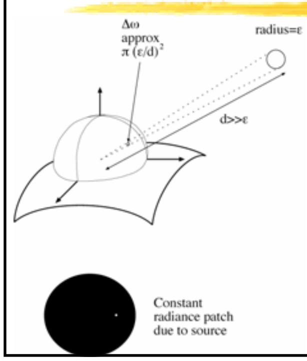
Exitance of a source: Internally generated power radiated per unit area on the radiating surface

A source can have both

- radiosity, because it reflects,
- exitance, because it emits.

$$B(P) = E(P) + \int \begin{Bmatrix} \text{Radiosity due to} \\ \text{incoming radiance} \end{Bmatrix} d\omega$$

POINT LIGHT SOURCE



Small, distant sphere radius ε and exitance E, which is far away subtends solid angle of about

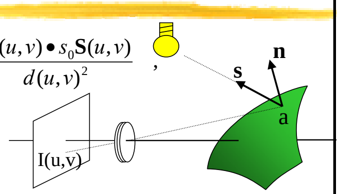
$$\pi\left(\frac{\varepsilon}{d}\right)^2$$

RADIOSITY DUE TO A POINT SOURCE

$$\begin{aligned}
B(P) &= \pi L_o(P) \\
&= \rho_d(P)\int_\Omega L_i(P,\omega)\cos\theta_i d\omega \\
&= \rho_d(P)\int_D L_i(P,\omega)\cos\theta_i d\omega \\
&\approx \rho_d(P)(\text{solid angle})(\text{Exitance term})\cos\theta_i \\
&= \frac{\rho_d(P)\cos\theta_i}{r(x)^2}(\text{Exitance term and some constants})
\end{aligned}$$

NEARBY SOURCE

Intensity $I(u,v) = \rho_d(u,v)\dfrac{\mathbf{N}(u,v) \bullet s_0\mathbf{S}(u,v)}{d(u,v)^2}$,



with

$\rho_d(u,v)$: Albedo of the surface projecting at (u,v).

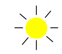
$\mathbf{N}(u,v)$: Direction of the surface normal.

s_0 : Light source intensity ε^2E.

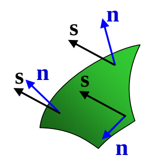
$\mathbf{S}(u,v)$: Direction to the light source.

$d(u,v)$: Distance to light source.

DISTANT LIGHT SOURCE



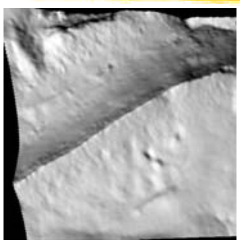
For light sources that are sufficiently far away from an object, all incoming rays are parallel.

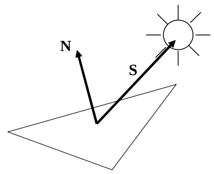


The **S** vector and the light source distance are taken to be *constant*.

→Lambertian Shading Model

LAMBERTIAN SURFACE

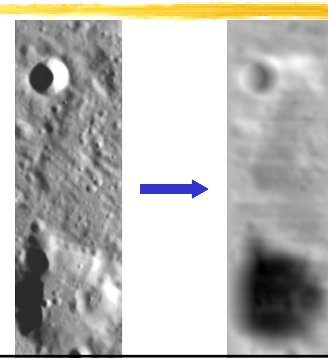




$I = \alpha(\mathbf{N} \bullet \mathbf{S})$

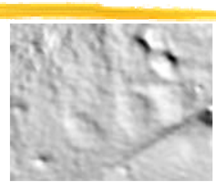
Perfectly matte surface: Radiance depends only on angle of incidence and not on viewing direction.

MOONSCAPE



LANDSCAPE

=



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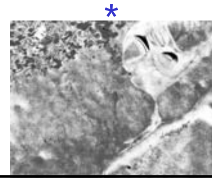
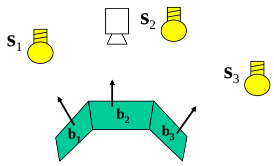


Q.T. Luong, P. Fua, and Y. G. Leclerc. The radiometry of multiple images. IEEE Transactions on Pattern Analysis and Machine Intelligence, 24(1):19--33, January 2002.

FACE

M. Dimitrijevic, S. Ilic, and P. Fua. Accurate Face Models from Uncalibrated and Ill-Lit Video Sequences. In Conference on Computer Vision and Pattern Recognition, Washington, DC, June 2004.

PHOTOMETRIC STEREO



Given multiple images of the same surface under different known lighting conditions, can we recover the surface shape?

Yes! (Woodham, 1978)

ALGEBRAIC FORMULATION

Lambertian model: $I = \alpha(\mathbf{N} \bullet \mathbf{S}) = \mathbf{S} \bullet \mathbf{M}$

Three light sources:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \mathbf{M}$$

$$\mathbf{N} = \frac{\mathbf{M}}{\|\mathbf{M}\|}$$

$$\alpha = \|\mathbf{M}\|$$

ADDITIONAL LIGHTS

Over-constrained problem:

$$\mathbf{I} = \mathbf{LM}, \text{ with } \mathbf{I} = \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} \text{ and } \mathbf{L} = \begin{bmatrix} \mathbf{S}_1 \\ \vdots \\ \mathbf{S}_n \end{bmatrix}$$

$$\Rightarrow \mathbf{L}^t\mathbf{LM} = \mathbf{L}^t\mathbf{I} \text{ (Least-squares solution)}$$

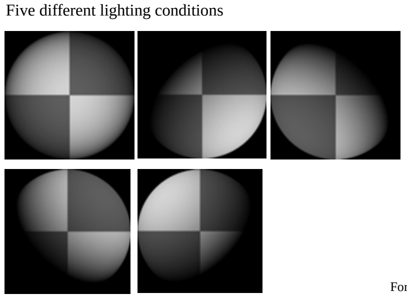
SHADOWS

- Shadowed pixels for a given light source position are outliers.
- Premultiplying by a thresholded weight matrix eliminates their contributions.

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \mathbf{I} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \mathbf{LM}$$

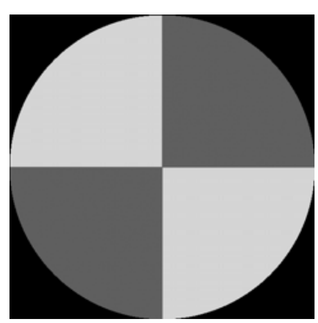
SYNTHETIC SPHERE IMAGES

Five different lighting conditions

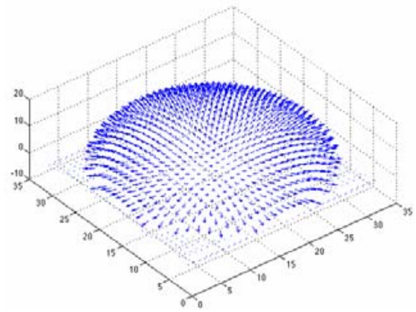


Forsyth & Ponce

RECOVERED ALBEDO



RECOVERED SURFACE NORMALS



SCANNING THE PIETA

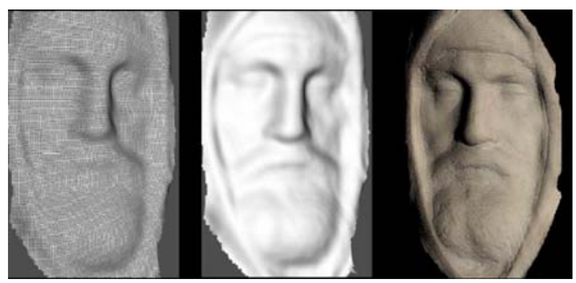
Recover both geometry and color from a set of images.

→ Build a 3—D model than can be used for virtual tourism.

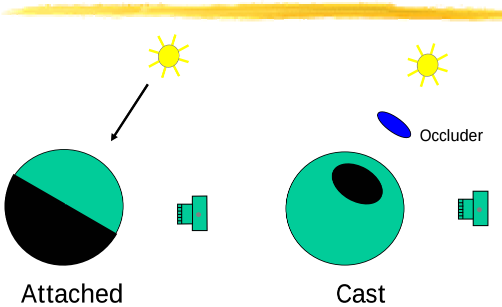
VIRTUOSO

One camera and five light sources

DELIGHTED TEXTURE MAPS

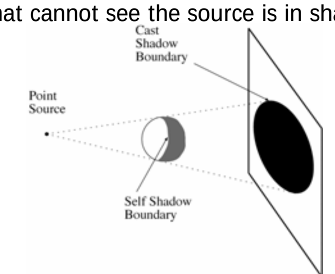


SHADOWS CAST BY POINT SOURCES



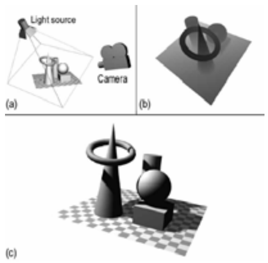
SHADOW GEOMETRY

A point that cannot see the source is in shadow.



→Shadow computation can be formulated in terms of visibility.

Z BUFFER



- Occlusion handling
- Creating shadows
- Voxelization
- ....

The Magic of the Z-Buffer: A survey. T. Theoaris et al. 2001.

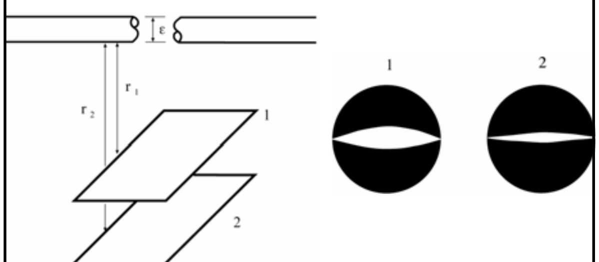
AMBIENT ILLUMINATION

Add a constant to the radiosity at every point in the scene to account for brighter shadows than predicted by point source model

Advantages: simple, easily managed

Disadvantages: poor approximation

LINE SOURCES



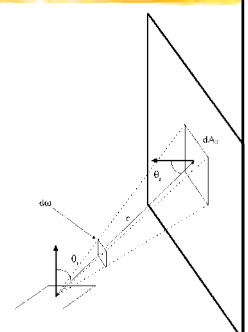
Radiosity due to line source varies with inverse distance, if the source is long enough.

AREA SOURCES

Examples:

- Diffuser boxes.
- White walls.

The radiosity at a point due to an area source is obtained by adding up the contribution over the view hemisphere section subtended by the source.



RADIOSITY DUE TO AN AREA SOURCE

$$B(x) = \rho_d(x)\int_\Omega L_i(x, u \to x)\cos\theta_i d\omega$$

$$= \rho_d(x)\int_\Omega L_e(x, u \to x)\cos\theta_i d\omega$$

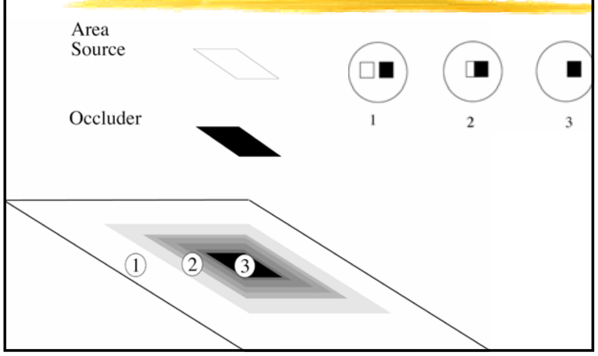
$$= \rho_d(x)\int_\Omega \left(\frac{E(u)}{\pi}\right)\cos\theta_i d\omega$$

$$= \rho_d(x)\int_{source} \left(\frac{E(u)}{\pi}\right)\cos\theta_i \left(\cos\theta_s \frac{dA_u}{r(x,u)^2}\right)$$

$$= \rho_d(x)\int_{source} E(u)\frac{\cos\theta_i\cos\theta_s}{\pi r(x,u)^2} dA_u$$

where x is a point on the surface and u a point on the source.

SHADOWS CAST BY AN AREA POINT SOURCE



EXPERIMENTAL FACT

- Prepare two rooms, one with white walls and white objects, one with black walls and black objects.
- Illuminate the black room with bright light, the white room with dim light
- People can tell which is which.

Why?

Gilchrist, Scientific American, 1979

BLACK ROOM UNDER BRIGHT LIGHT

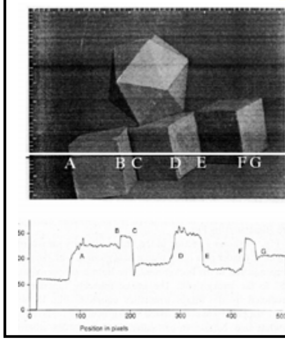


Image intensity along the line drawn in the image.

Forsyth and Zisserman, CVPR 1989

WHITE ROOM UNDER DIM LIGHT

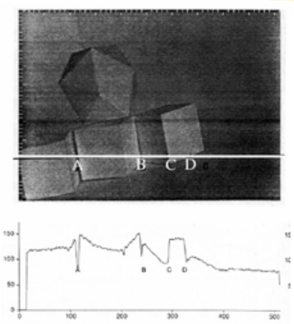


Image intensity along the line drawn in the image.

Forsyth and Zisserman, CVPR 1989

EXPERIMENTAL FACT

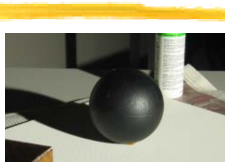
- Prepare two rooms, one with white walls and white objects, one with black walls and black objects.
- Illuminate the black room with bright light, the white room with dim light
- People can tell which is which.

Why: Because of interreflections!

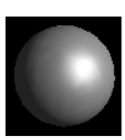
Gilchrist, Scientific American, 1979

SECONDARY ILLUMINATION

Reflections produce indirect lighting.



Unique light source assumption doe not allow correct albedo recovery.

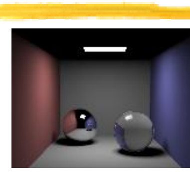


Recovered shading

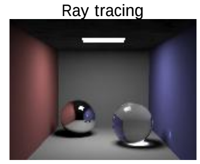
Recovered albedoes

SYNTHETIC BALLS

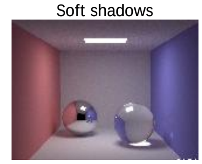
Ray tracing



Soft shadows

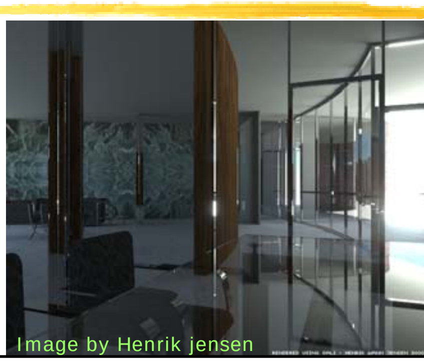


Interreflections

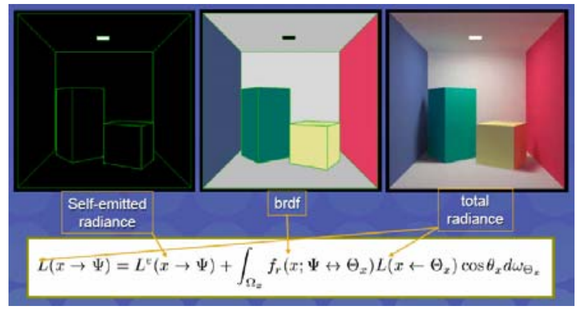


Ambient lighting

GLOBAL SHADING MODELS

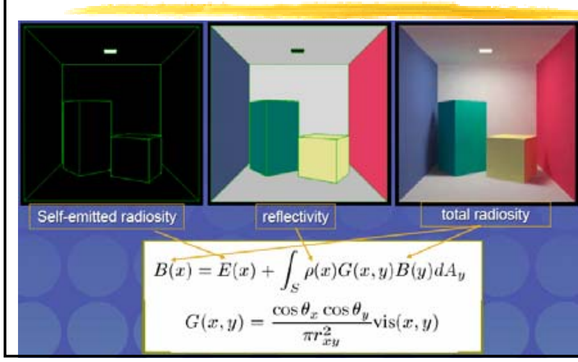


ILLUMINATION EQUATION

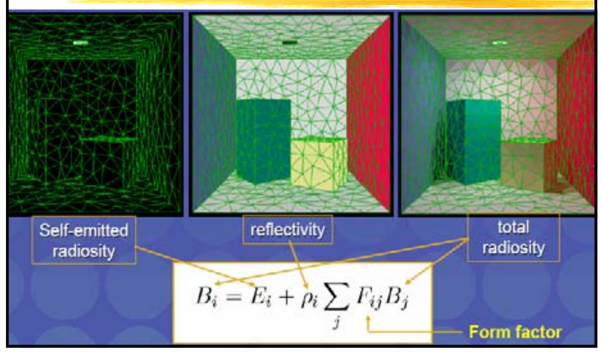


P. Bekaert, PhD Thesis, K.U. Leuven 1999

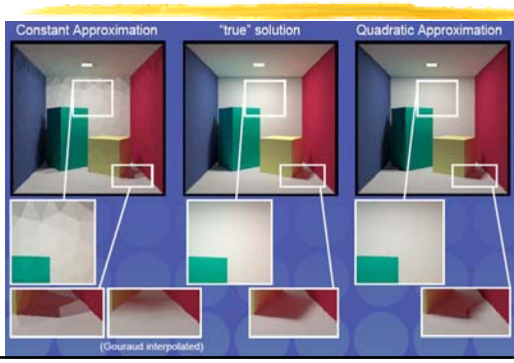
USING ONLY DIFFUSE MODELS



LINEARIZATION



ARTIFACTS



IN SHORT

Local shading models:

- Simple to formulate
- Well adapted for vision style computations
- Unrealistic

Global shading models:

- Much more complex
- No obvious way to use them for analysis
- Yield realistic synthesis results